PHYSICS 202 LAB 3: THE LOOP RULE AND THE JUNCTION RULE

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THEORETICAL DISCUSSION

Two very important rules for analyzing the behavior of circuits are the Loop Rule and the Junction Rule. These are often alternately referred to as Kirchoff's 1st and 2nd Laws. They are not actually independent laws or rules, in the sense that Conservation of Momentum, for example, but instead they are manifestations of other more fundamental rules.

The Loop Rule

The Loop Rule states that the total change in potential around a closed loop is zero. This is just a manifestation of the Conservation of Energy. Mathematically we can write this as

$$\sum_{j=1}^{N} \Delta V_j = 0 \qquad \textbf{Eqn. 1}$$

In the circuit shown in Figure 1, the Loop Rule states that

$$V_1 - IR_1 - V_2 - IR_2 = 0$$

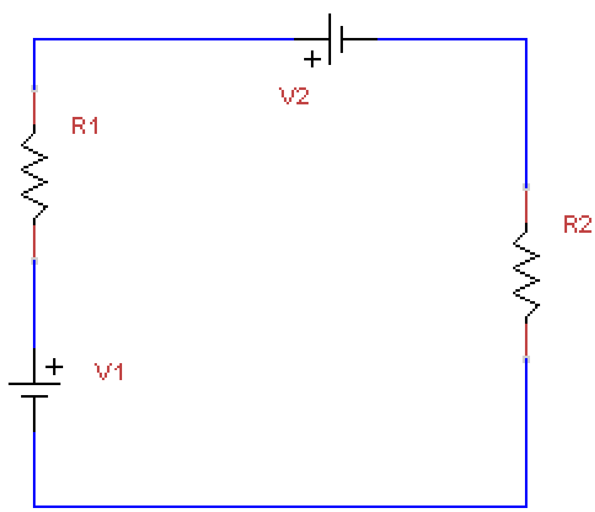

Fig. 1: Loop Rule Example

In implementing the Loop Rule, we subtract potential V_2 rather than add it because in going clockwise around the loop, you cross V_2 from positive to negative.
A more complex circuit, such as that shown in Figure 2, has two independent loops. Note that when you cross a resistor, the change in potential is negative if you cross in the direction of the current and positive if you cross in a direction opposite to the current.

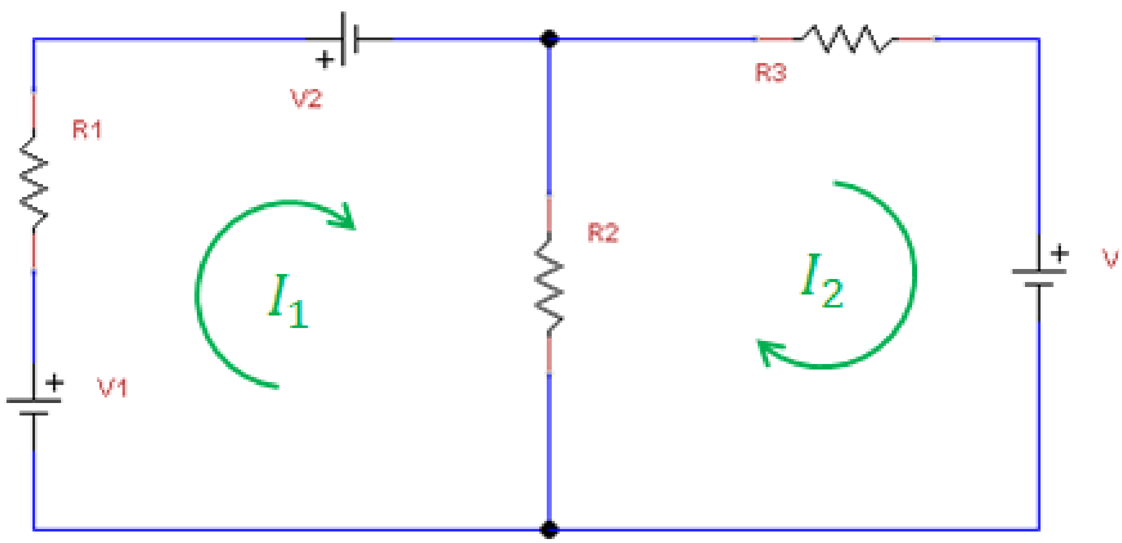

Fig. 2: A Two Loop Circuit

This circuit leads to the following loop equations:

$$V_1 - I_1R_1 - V_2 - (I_1 - I_2)R_2 = 0$$

$$-V_3 - (I_2 - I_1)R_2 - I_2R_3 = 0$$

$$V_1 - I_1R_1 - V_2 - I_2R_3 - V_3 = 0$$

Only two of these three equations are independent. The other one is equal to the sum or difference of the other two.

The Junction Rule

The Junction Rule states that the current entering a junction is equal to the current leaving a junction. This is a manifestation of the Conservation of Current. If the currents entering a junction are taken as positive, and currents leaving a junction are taken as negative, then a succinct mathematical statement of the junction rule is

$$\sum_{j=1}^{N} I_j = 0 \qquad \textbf{Eqn. 2}$$

Where the sum is taken at the junction.

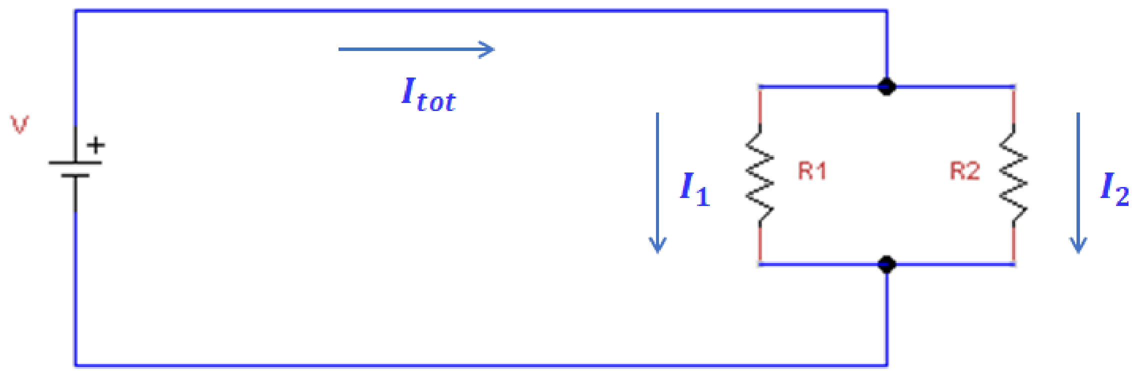

Fig. 3: Junction Rule Example

The mathematical relation between the currents is thus

$$I_{tot} = I_1 + I_2$$

Using the Loop Rule, we can see that $I_1 R_1 = I_2 R_2 = V$. Thus, $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, and so the total current is

$$I_{tot} = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_1} = V\left(\frac{1}{R_1} + \frac{1}{R_1}\right)$$

Experimental Procedure

Build the circuit shown in Figure 4.

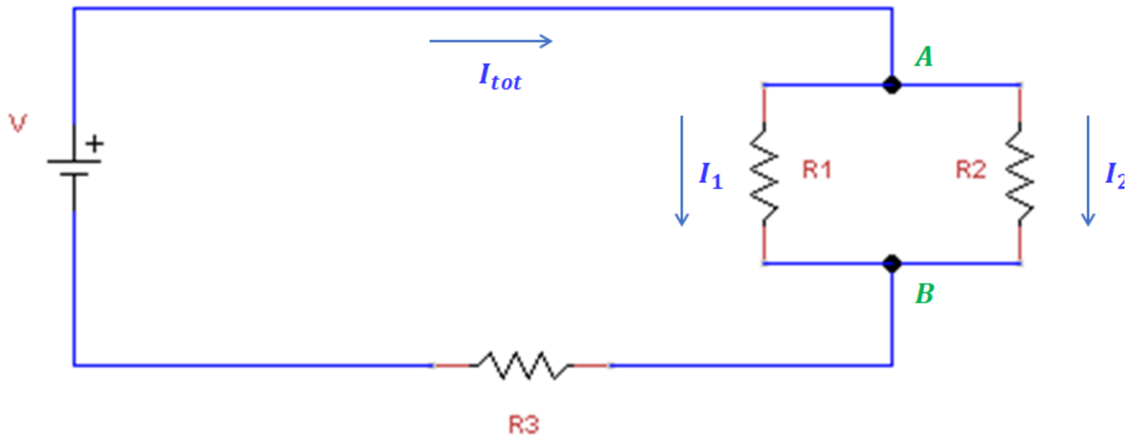

Fig. 4: Experimental Circuit

1. Measure and report V, I_{tot}, I_1, and I_2.

2. Measure and report V_{AB}; the potential drop between points A and B in the circuit.

3. Measure and report V_3; the potential drop across resistor R_3.

4. Calculate and report $I_{sum} = I_1 + I_2$.

5. Calculate and report $\delta_{cur} = \left|\frac{2(I_{sum}-I_{tot})}{(I_{sum}+I_{tot})}\right|$. This is the (dimensionless) measure of the fractional discrepancy between the predicted and measured total current.

6. Calculate and report $V_{sum} = V_{AB} + V_3$.

7. Calculate and report $\delta_V = \left|\frac{2(V-V_{sum})}{(V+V_{sum})}\right|$. This is the (dimensionless) measure of the fractional discrepancy between the predicted and total potential.