RD Instruments

WAVES PRIMER:

Wave Measurements and the RDI ADCP Waves Array Technique

RD Instruments Inc.
9855 Businesspark Avenue
San Diego CA 92131
Tel: +1 858-693-1178
Fax: +1 858-695-1459
Email: sales@rdinstruments.com
Web: www.rdinstruments.com

Introduction:

Acoustic Doppler Current Profilers (ADCPs) gather profiles of water velocity by measuring the Doppler shift of sound reflected from scatterers assumed to be passively following the flow. The measurements are range-gated into a series of bins along three or more beams and then combined to infer the velocity profile encompassed by the beams. This technology is well-proven, and ADCPs are now routinely deployed around the world.

Directional wave measurements, by whatever technique, seek to statistically describe basic wave parameters in terms of the wave amplitude, period and direction. The wave amplitude measurement most commonly employed is known as the Significant Wave Height, H_s; loosely considered to be the average peak-to-peak amplitude of the largest one third of the waves seen during the measurement interval. The peak period, T_p, tells the characteristic frequency of the arriving wave energy (frequency is the inverse of the period). The mean direction of the waves, D_p, tells which way the waves are propagating. To summarize, directional wave measurements tell how much wave energy exists (H_s), at what frequency (T_p) and from what direction (D_p).

Wave Basics

It is perhaps worthwhile to go over some of the fundamental behaviors of ocean waves. To make the process as simple as possible, consider a single wave propagating in the *x* direction. The equations that describe this wave are presented without derivation (see Kundu's 1990 text for an excellent presentation of the derivation):

$$u = a\omega \frac{\cosh k(z+H)}{\sinh kH} \cos(kx - \omega t) \tag{1}$$

$$w = a\omega \frac{\sinh k(z+H)}{\sinh kH} \sin(kx - \omega t) \tag{2}$$

$$\omega = \sqrt{gk \tanh kH} \tag{3}$$

Where: u is the horizontal velocity of a parcel of water
w is the vertical velocity of a parcel of water
a is the amplitude of the wave at the surface
ω is the angular frequency of the wave (which is $2\pi/\tau$ where τ is the period)
k is the wave number of the wave (which is $2\pi/\lambda$ where λ is the wavelength)
H is the water depth
g is acceleration due to gravity

Several important features of ocean waves can be seen from a careful inspection of these equations:

- Equations (1) and (2) show that the amplitude of the velocity fluctuations of a parcel affected by the passage of the wave depends on its depth (z), the total water depth (H), angular frequency (ω), and wave number (k).
- Equations (1) and (2) also show that the amplitude of the horizontal velocity fluctuations (u) is subject to different constraints than the amplitude of the vertical velocity fluctuations (w).

- Equation (3) shows that the angular frequency (ω) is related to both the wave number (k) and the water depth (H). Frequency generally does not change, so a wave of a given frequency that propagates into shallower water will change its wave number, and hence its wavelength.

Some of the interplay between these parameters can be seen with an illustration of a wave propagating into shallower water. Waves are often characterized by their period, and an eight second wave is shown.

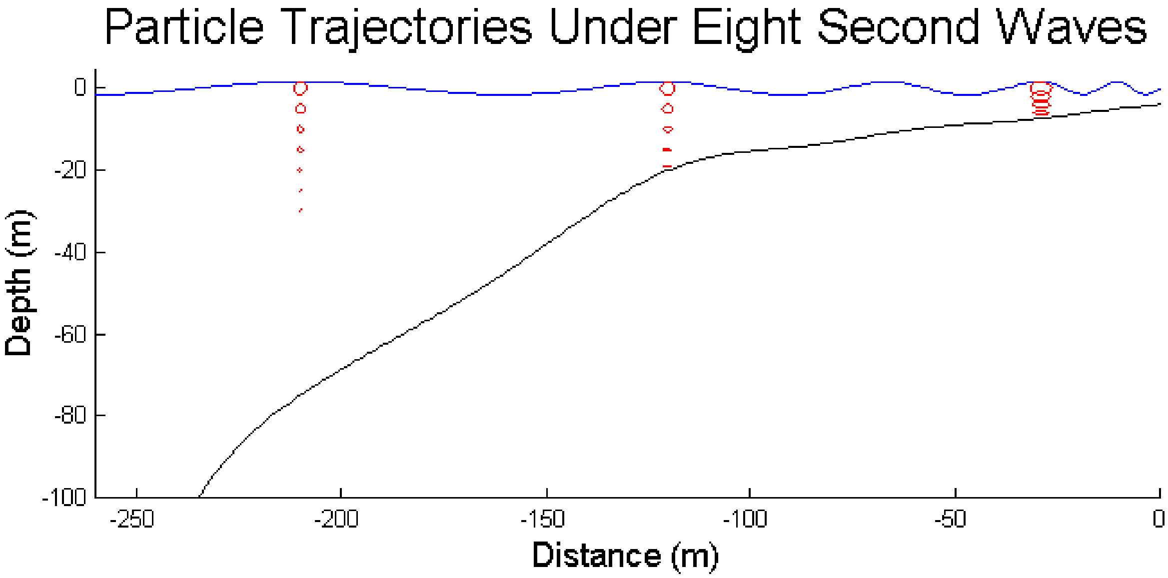

Fig. 1: A two meter wave with an eight second period is shown propagating into shallower water. Each red trace represents a particle's path as the wave passes. Three cases are shown: deep water, intermediate water and shallow water.

In deeper water, the path followed by any parcel of water affected by the wave is a circle whose diameter decreases with depth. This is important because it means the wave energy only propagates to some finite depth, beneath which it can not be seen (or measured). As the wave begins to "feel" the bottom the vertical velocity of the parcel attenuates more rapidly with depth than does the horizontal velocity until, as shown in the shallowest example, the vertical attenuation is so much stronger than the horizontal attenuation that the parcels near the bottom track an entirely horizontal path. It is very important that this depth dependent behavior of waves be understood because the intent is to measure the wave's characteristic properties with a subsurface instrument. Note that the wavelength shortens as the wave propagates into shallower water.

Now consider a shorter period wave:

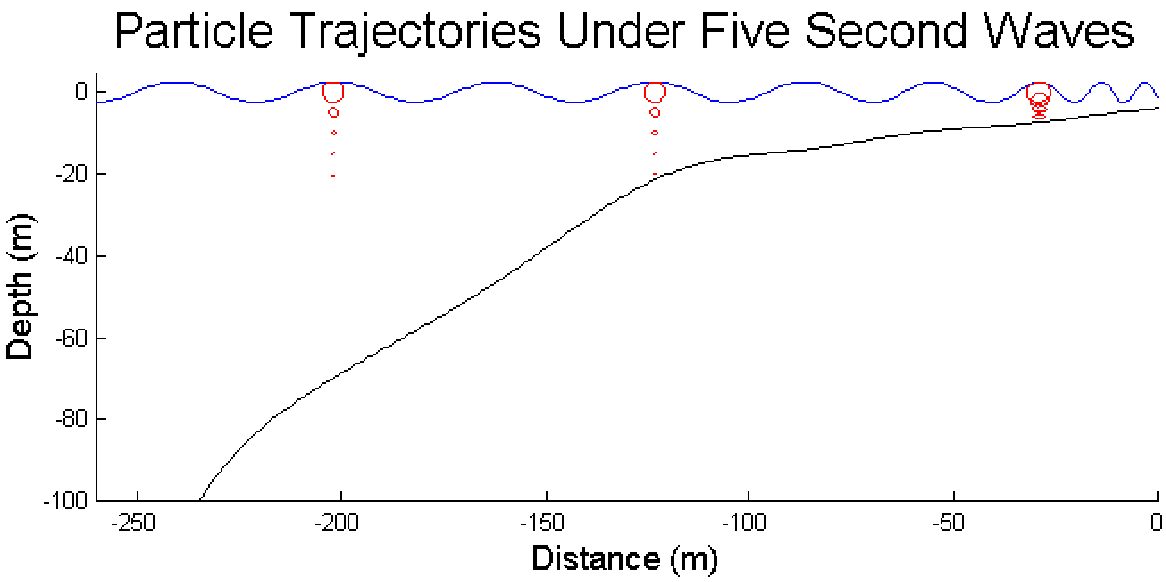

Fig. 2: A two meter wave with five second period is shown identically as in Fig. 1.

Notice that the five second wave behaves substantially differently than the eight second wave. First of all, the amplitude of the parcel excursions decreases with depth more quickly than the eight second wave. This is very important because it shows that higher frequency waves do not penetrate as deeply into the water as lower frequency waves, which in turn means that the frequency of waves that can be measured depends on the depth of the measurement. Because this five second wave is higher frequency than the eight second wave in Fig. 1 the five second wave is not yet "seeing" the bottom at 20 m depth, and the particle trajectories there are identical to those in deeper water.

The approximate decay of measurable wave periods with depth is shown in Fig.3.

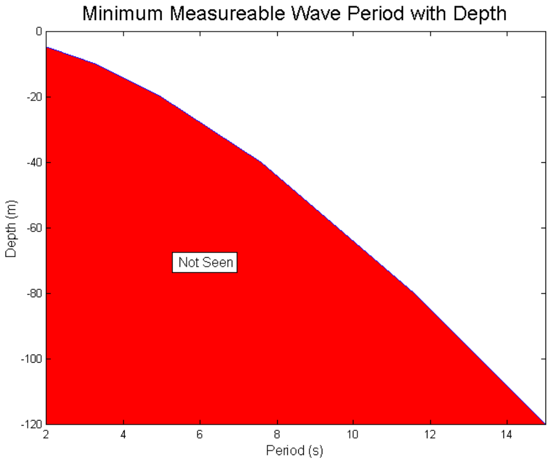

Fig. 3: The approximate decay of wave energy with depth. For example, waves of period shorter than five seconds do not penetrate below 20 m depth.

A Statistical Description

Unfortunately, the real ocean is rarely if ever made up of a single wave. Almost any sea that is carefully watched can be maddeningly irregular. However, there are some very powerful tools readily available if a few basic assumptions are postulated. First is the assumption that the wave field can be described as the summation of sine waves of varying frequency, amplitude and direction. Second is the assumption that the field is statistically stationary – which means that the statistical description of the waves at a given time is essentially the same description that would be obtained at a slightly different time.

Joseph Fourier was a French mathematician who first proposed (in 1807) the idea that any periodic function could be represented as a summation of sine waves. While this idea was initially greeted with outrage, it is safe to say that his ideas have since become one of the primary tools of the physical sciences, and a full exposition is far beyond the scope if this primer. Suffice to say here that Fourier analysis allows a means to reduce a measured time series to a few constituent sine waves, which reveal the dominant frequency components of the process being measured.

Power Spectra

In the plots that follow, the time series of water level is shown in the top panel, and its power spectral density plot is shown in the bottom panel. The power spectral density plot shows how the power is distributed between sine waves of varying frequency. If the entire field consists of a single sine wave, then the power spectral density will be sharply peaked around that frequency. The units for Power Spectral Density are decibels (dB), which is a logarithmic scale – so a decrease of three decibels shows a halving of the power. A 60 dB decrease corresponds to a decrease in power by a factor of one million.

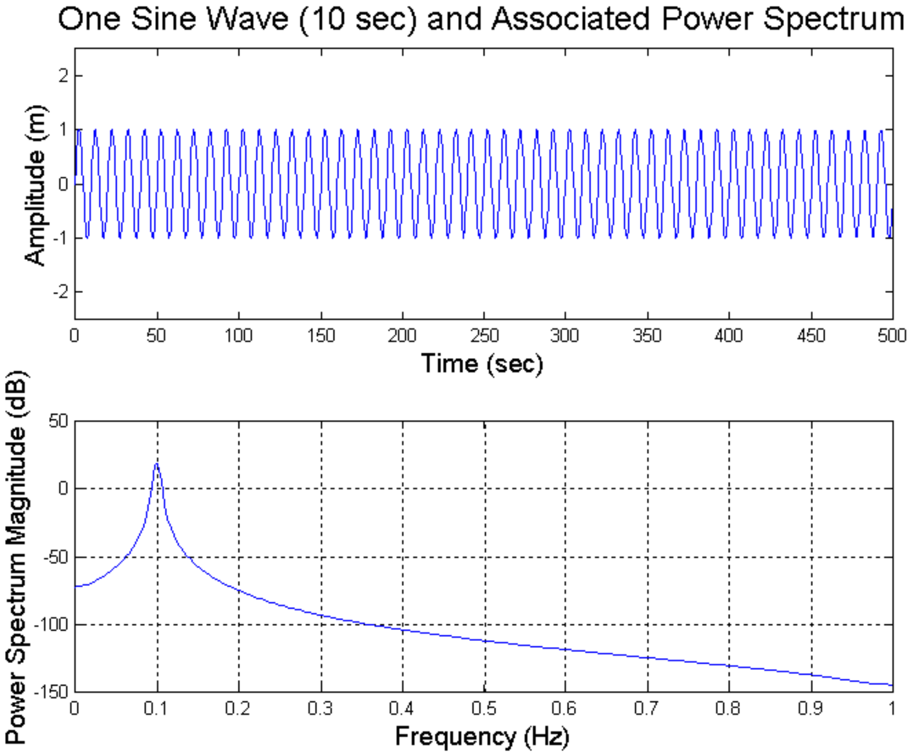

Fig. 4: The power spectral density of a single sine wave with ten second period shows that the energy is concentrated at ten seconds (0.1 Hz).

If the field is made up of two sine waves then the power spectral density will show the energy distributed between the two peaks. If the two sine waves are of equal amplitude, then the two peaks of the power spectral density will be of the same height. It is interesting to note (Fig. 5) that the time series created by the addition of two sine waves already seems fairly complicated, while the power spectral density reveals that it was created quite simply.

Side note: In the example shown in Fig. 5, the time series was created by summing two sine waves of similar frequency. It can be shown with basic trigonometry that the sum of two sine waves of similar frequency will result in a function which oscillates at the average frequency, but whose amplitude is bounded by a sine wave whose frequency is the difference of the two frequencies. This phenomenon is known as “beating”, and it is a very common feature of real ocean waves. Any beach observer knows that waves tend to occur in fairly regular “sets” of large and small amplitude, which is readily attributable to the effect of adding waves of similar frequency as shown in Fig. 5.

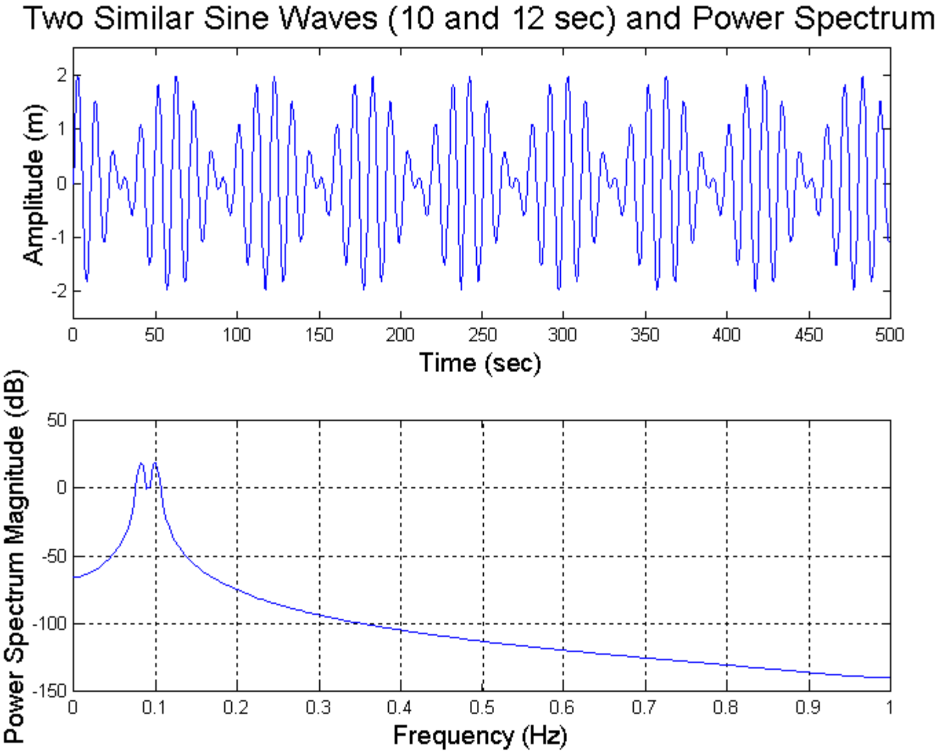

Fig. 5: The power spectral density plot reveals that the complicated time series shown is made up entirely of only two sine waves of similar frequency and equal amplitude.

Now, consider a "real" ocean measurement:

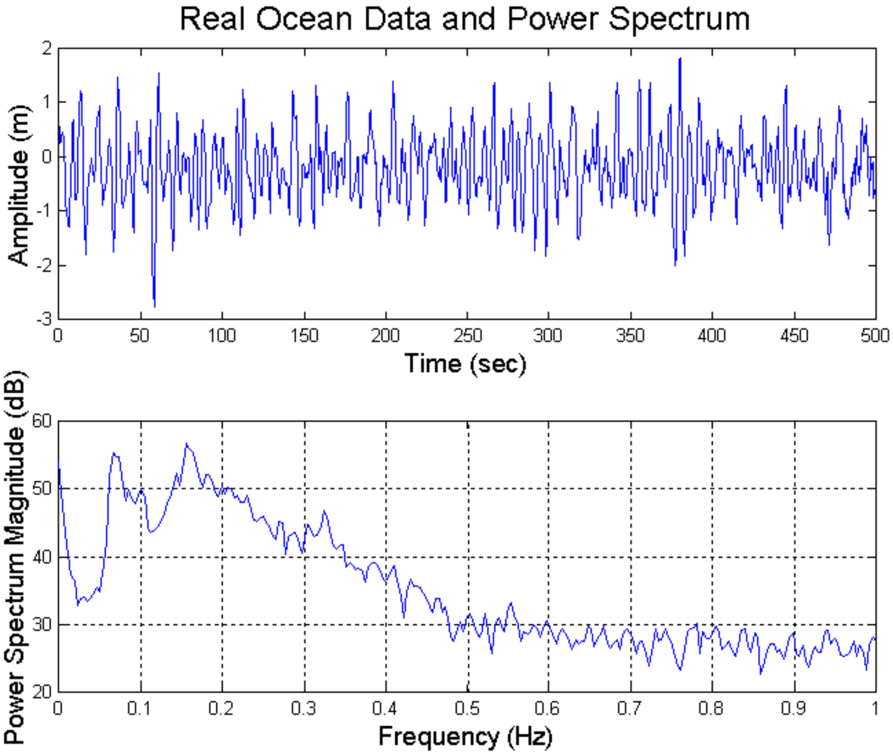

Fig. 6: The power spectral density plot of a real ocean measurement shows that the seemingly hopelessly complicated time series is in fact dominated by only two waves with periods of about 6 and 14 seconds.

The power spectrum of the real ocean contains energy at all frequencies, rather than the artificial constructs shown above, which have all energy concentrated at the set frequencies. This spectrum clearly shows that the bulk of the energy in this very complicated time series is contained in two frequencies of about 0.07 and 0.16 Hz.

Cross Spectra

Cross Spectra provide a relatively straightforward means to compare whether two different measured parameters are varying together. If two different measured parameters are varying at the same frequency, then it is likely that they are related. An obvious example is to compare the water level and velocity of a simple plane wave (Fig. 7).

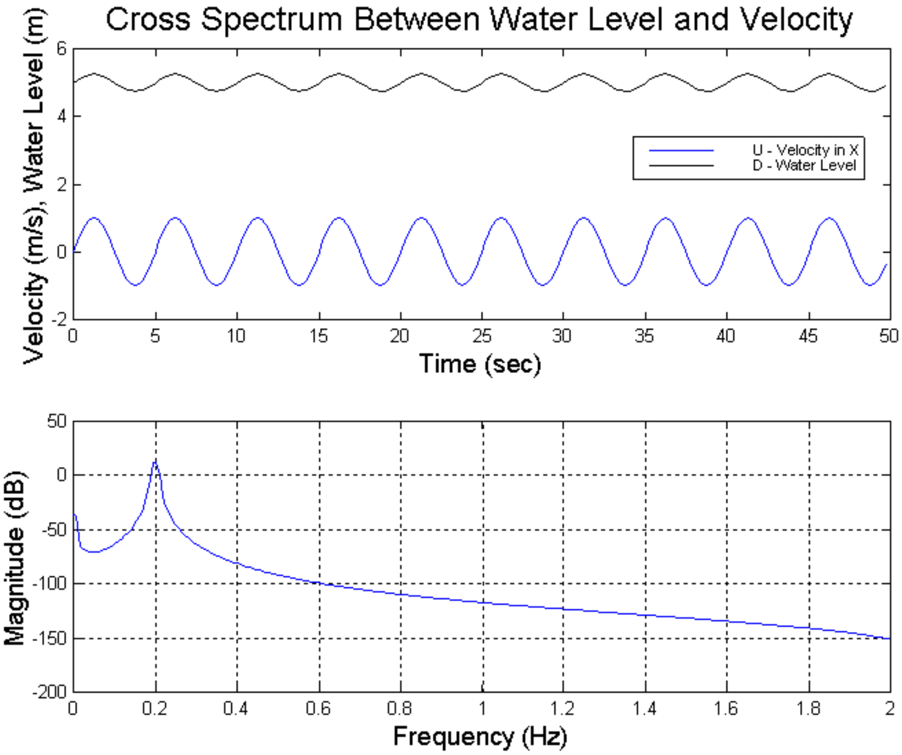

Fig. 7: The time series of water level and horizontal velocity are displayed in the top panel, and their cross spectral density is plotted in the lower panel. It is clear that water level and horizontal velocity are both varying at the same frequency (0.2 Hz)

If we complicate the water level measurement with a second sine wave that is not present in the horizontal velocity (because the second wave is propagating perpendicularly to *U*), then we see (Fig. 8) that the cross spectrum of *U* with *D* shows tremendous energy at 0.2 Hz, and only a slight bump at a second frequency.

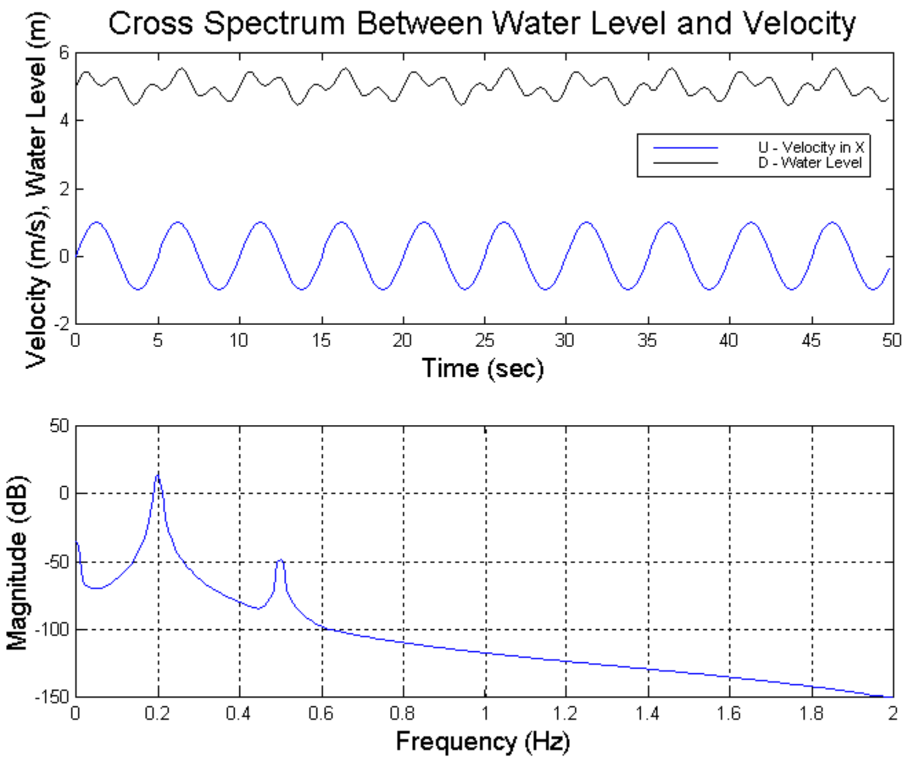

Fig. 8: As in Fig 7., but a second wave has been added that is propagating perpendicular to the measured velocity.

The slight bump at the other frequency can be neglected because it is so much lower in power (60 dB corresponds to one million times less power). It is present because the modeled water level is made up of several thousand measurements of two simple sine waves with different periods (2 and 5 seconds), so there will be a subset of measurements (every ten seconds) where the water level seems to vary with *U* at the other frequency. That the power in this frequency is so much lower is the tip-off that it is not real – or at least not important.

Wave Parameter Measurements

It is useful to separate a discussion of the wave field into two parts: the parameters that do not depend on the direction of the wave propagation, H_s and T_p; and the parameter that does, D_p.

Non-Directional Wave Parameter Measurement

Peak Period (T_p)

Wave frequency does not change with depth or direction, making peak period the easiest to measure of the three commonly reported wave parameters. A time series of any of a number of parameters that vary with the wave frequency is all that is required. Such parameters include water level, pressure and the orbital velocity of the passing waves. Methods for directly measuring water level include capacitance wave gauges, laser altimeters (looking down in air) and inverted echo sounders (looking up in water). Subsurface instruments can be used to measure pressure and/or orbital velocities, but it is important to remember that the higher frequency waves may not penetrate to the measurement depth. The practical effect of this is that a tradeoff exists between the depth of the measurement location and the ability to measure (or even see) the higher frequency waves. Once the time series of the varying parameter is obtained, its power spectral density function will reveal the frequency with the most energy, and hence the peak period.

Significant Wave Height (H_s)

The time series of the water level can be obtained directly by a number of means, including capacitance wave gauges, laser altimeters (looking down in air) and inverted echo sounders (looking up in water). It is also quite common to measure the pressure and/or orbital velocities from subsurface instruments. However, these parameters decay with depth as described above and therefore must be transferred to their equivalent surface values using equations (1)-(3) for the velocities, and a similar equation for the pressure.

Significant wave height is the average peak-to-peak amplitude of the largest one third of the waves in a given field. This value is roughly equivalent to four times the square root of the value obtained by integrating the non-directional spectrum from the time series of the surface level, and in practice it is usually this integration that is actually reported as the significant wave height. There are a couple of practical issues that arise from calculating the significant wave height in this manner:

1) Spectra from the real ocean are quite noisy, so in practice it is common to band average the frequency spectrum – that is, reduce the spectrum to frequency bands whose reported energy is the mean energy within the band.
2) In any real measurement there is a noise floor, that is, a lower bound that the spectrum asymptotically approaches. This is readily seen in Fig. 5 where the spectrum is seen to "flatten out" to 25 dB or so at about 0.6 Hz. This noise has several sources, including: that the measurement is not perfect, that the instrument itself will have noise, turbulent fluctuations of the water, etc. This noise floor is usually removed before calculating significant wave height because including it will bias the calculation high.
3) The high and low frequency cutoffs of the spectrum can vary due to the sampling and the noise levels. It is important that the main peak of the characteristic wave energy is located well within the high and low frequency cutoffs or some of the energy is "lost" which will result in an underestimation of the significant wave height. For example, in Fig. 9 below we show the spectrum from Fig 4 (a single sine wave of ten second period). If we set the low frequency end of our spectrum too high, say eleven seconds here, a significant portion of the wave energy is actually at frequencies lower than we include in the integration, and our significant wave height will be underestimated.

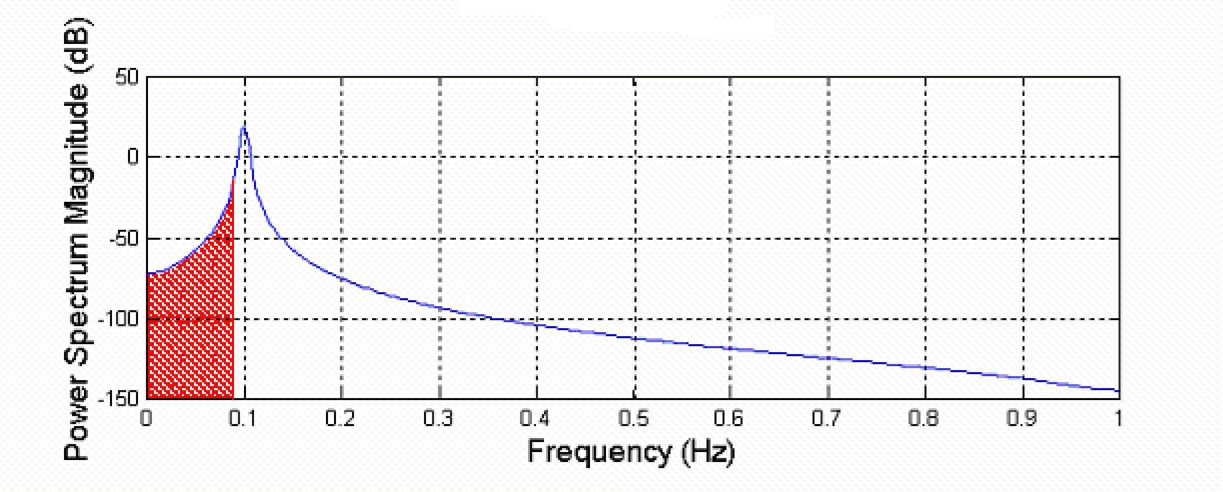

Fig. 9: The power spectral density function as generated in Fig. 4. If we set the low frequency cutoff to eleven seconds, then all of the areas marked with red cross-hatches would be neglected in our significant wave height calculation. Since this spectrum was generated by a wave of ten seconds, quite near our cutoff, excluding the marked area will result in an underestimate of the significant wave height.

The above three points apply to whatever method is used to measure the time series of water level. There is an additional consideration that must be made when the water level measurement is inferred from a subsurface velocity or pressure measurement, and that is the effect of background currents on those measurements. Fig. 10 shows the same eight second wave as was shown in Fig. 1, but an onshore background current of 0.75 cm/s has been added.

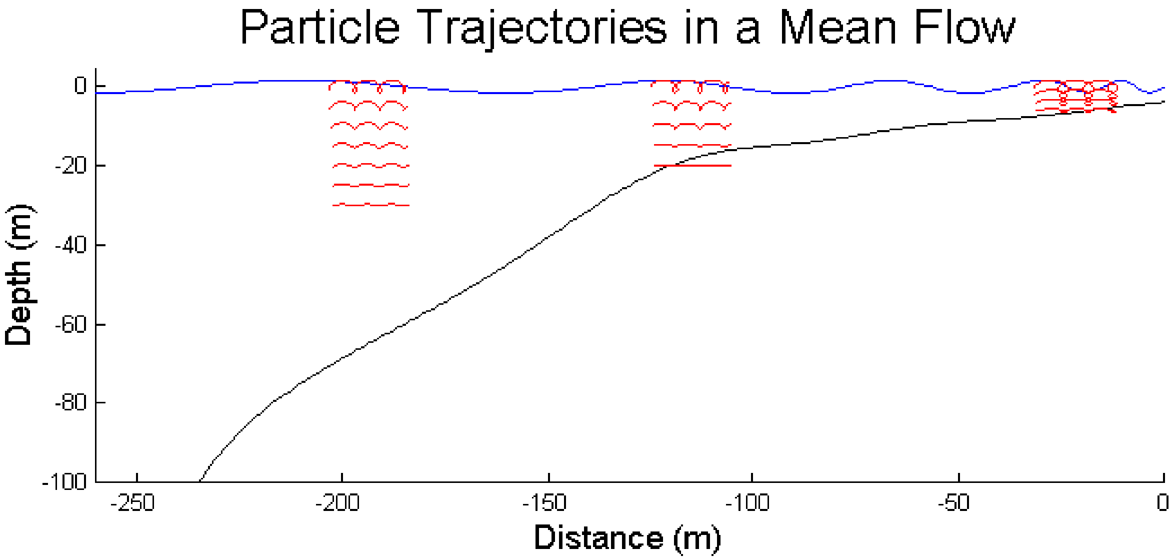

Fig. 10: An eight second wave as in Fig. 1, but with an onshore background current of 0.75 cm/s. Note the substantial elongation of the particle paths in comparison to Fig. 1.

The background current results in a substantial elongation of the particle paths. Any subsurface measurement of velocity (or pressure – any change in velocity is accompanied by a proportional change in pressure) will be larger than if the background current were not there. The subsurface measurement simply reports the total velocity or pressure; it is incapable of distinguishing the changes due to waves from other forcing. Since the subsurface measurements must be transferred to a surface level for wave height estimation, these artificially large measurements will result in an artificially large measurement of the water level, and therefore a reported significant wave height that is too high. The transfer to the surface is specifically correcting for an assumed decay in the velocity, so deeper measurements will be amplified more than shallower measurements. Hence, deeper measurements that do not correct for the presence of background currents will report higher significant wave heights than shallower measurements in the same background current.

This effect can be removed by including the effects of background currents in the dispersion relation (equation (3)) as follows:

$$\omega - kU\cos\alpha = \sqrt{gk\tanh kH} \qquad (4)$$

where U is the vertically weighted magnitude of the background current
α is the angle between the background current and the wave direction
all other variables are as in equation (3)

Note that equation (3) is recovered identically if the background current is zero. It is also worth noting that the appropriate velocity is the vertically weighted velocity, which requires a profile of the velocity through the water column.

Directional Wave Parameter Measurement

It is common practice to separate the directional wave spectrum into a frequency spectrum and a directional spreading function, as follows:

$$DW(\omega,\theta) = S(\omega)D(\omega,\theta) \tag{5}$$

The frequency spectrum *S(ω)* is the same non-directional spectrum used to determine parameters like the significant wave height and peak period as described in the previous section. There are a number of ways to estimate the directional spreading function, *D(ω,θ)*, most of which involve comparing the cross spectra between measurements of some number of independent parameters.

It is very common to compare measurements of pressure and horizontal velocity to determine the wave direction. This is known as the PUV method, which really boils down to answering a simple question: "Does the variation in water level (P) vary more with the velocity in the *x* direction (*U*) or the velocity in the *y* direction (*V*)?" Consider the case where the wave is propagating entirely in the *x* direction:

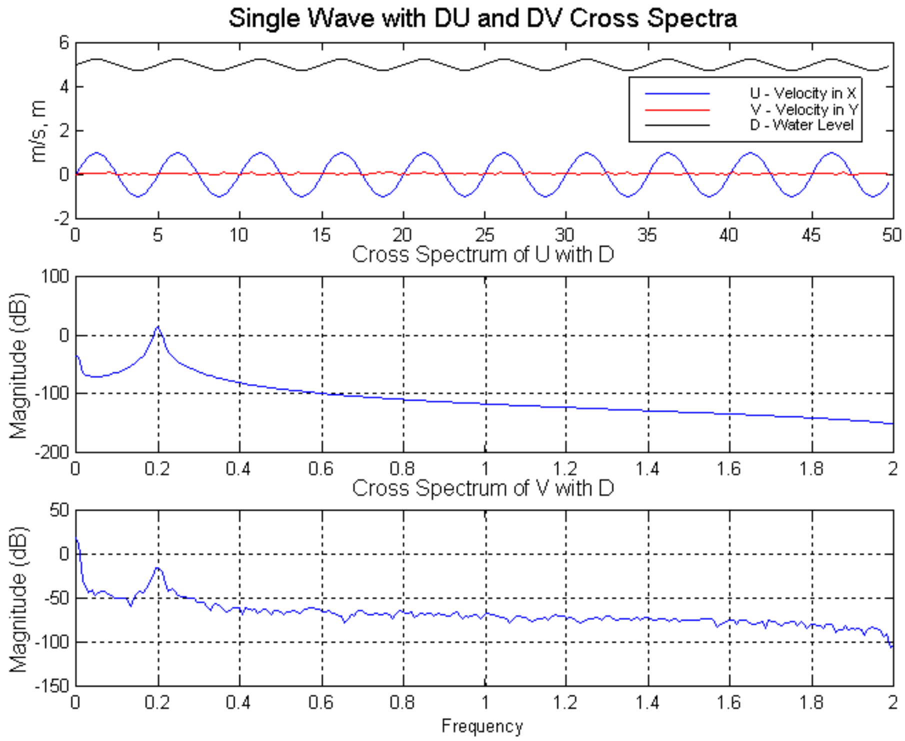

Fig. 11: A simple sine wave propagating in the x direction and the cross spectra between water level and each component of the horizontal velocity.

Both cross spectra in Fig. 11 show some energy at 0.2 Hz, but that there is far more energy in the *x* direction than in the *y* direction. In the PUV method the direction is obtained by comparing the magnitude of the cross spectra at each frequency (or at each averaged frequency band):

$$D_P(f) = \tan^{-1}(C_{DU}(f)/C_{DV}(f)) \tag{6}$$

Where C_{DV} is the cross spectrum of D with V
C_{DU} is the cross spectrum of D with U
f is the frequency (or frequency band) of interest

This technique will also work if waves of different frequency are propagating in different directions. Consider the case where one wave is propagating along the *x* direction and the other is propagating along the *y* direction.

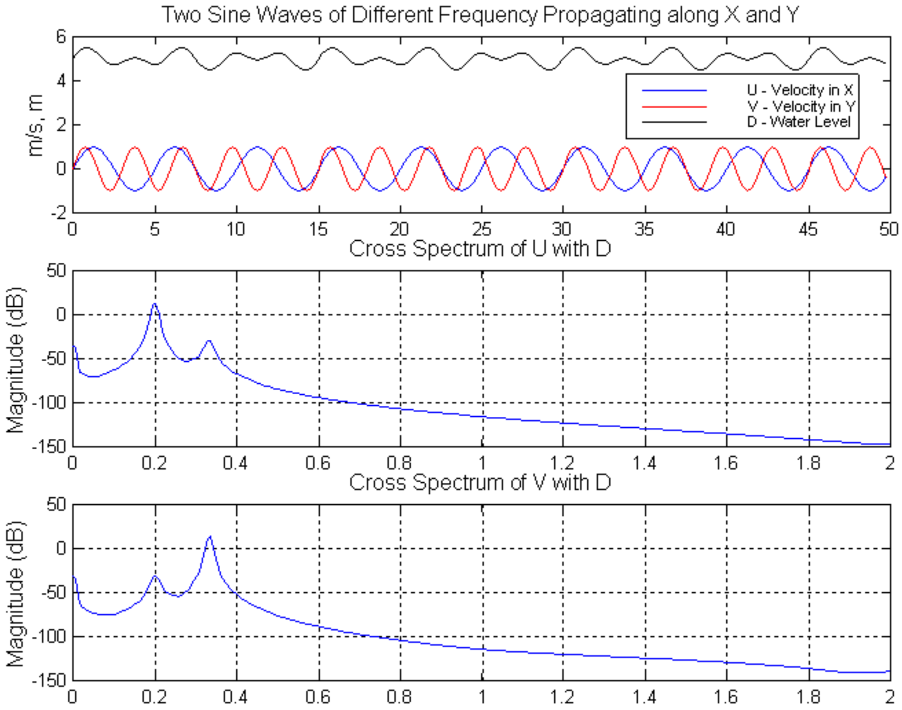

Fig. 12: The top panel shows the time series of horizontal velocity and water level for two waves of different frequency propagating in different directions. The middle panel is the cross spectral density of U with D and the bottom panel shows the cross spectral density of V with D.

The cross spectral density plots of Fig. 12 clearly recover the fact that our model consists of a five second wave propagating in the *x* direction (velocity *U*) and a three second wave propagating in the *y* direction (velocity *V*).

The PUV method has the primary advantage that a single, bottom-mounted instrument can provide all of the measurements necessary to make the calculations – making it very simple to deploy. There are a couple of significant limitations to the PUV technique however:

1) The PUV technique measures velocity at a single level. In the presence of a background current a weighted average of the background current over the full water column is needed to properly transfer the measurements to the surface using equation (4).
2) At any given frequency or frequency band, the PUV technique will report a single direction. This is a fundamental limitation to the technique that is attributable to the limited number of measurements that have been made. If there are two or more wave trains of similar frequency propagating in different directions across a PUV gauge, the PUV gauge will choose some intermediate direction which might not actually have any wave energy at all. This is important because crossing wave trains will result in different forcing, and different coastal processes, than a single wave train from an intermediate direction.

Arrays

In order to resolve a multidirectional wave distribution at a given frequency more measurements must be made, which generally means deploying an array of instruments. Making additional measurements from instruments deployed with specific spatial separation provides the additional information needed to resolve the full multidirectional wave field for each frequency. A typical setup might involve careful installation of several water level sensors (which could be pressure sensors, capacitance wave gauges, inverted echo sounders, laser altimeters, etc.) at precisely surveyed locations. The cross spectra of these measurements are then compared in any number of ways to yield the directional spreading function.

The cross spectrum of the elements in the array can be defined as:

$$C\omega) = \int H(\omega,\theta)D(\omega,\theta)H^{\dagger}(\omega,\theta)d\theta \tag{7}$$

Where *C(ω)* is the cross spectra of the measurements
H(ω,θ) is the array response to a uniform plane wave of unit amplitude propagating in direction *θ*
D(ω,θ) is the directional spreading function to solve for
$H^{\dagger}(\omega,\theta)$ is the Hermitean transpose of H

The fundamental limitation of this equation is that *C(ω)* is finite (limited by the number of elements in the array), while there are a multitude of solutions *D(ω,θ)* that will satisfy equation (7).

There are a number of proven approaches to solving this equation for the directional spreading function, only one is presented here. The Maximum Likelihood Method (MLM) assumes a solution at each frequency is of the form:

$$D(\theta) \propto \frac{1}{H(\theta)^{\dagger} C^{-1} H(\theta)} \tag{8}$$

Where C^{-1} is the inverse of *C*

In essence, equation (8) is solved for each frequency at each direction to determine if any wave energy exists at that frequency and in that direction. As long as *C* contains cross spectra from a sufficient number of independent measurements and *H* is well defined, then it is possible to resolve multiple wave trains of differing direction but similar frequency because the method specifically looks for waves in every direction at each frequency.

The RDI ADCP Waves Array Technique

The RD Instruments (RDI) ADCP Waves Array Technique combines the benefits of deploying a single, bottom mounted instrument with the multidirectional capabilities of an array. In addition, the ADCP provides the vertical velocity profile information needed to avoid overestimation of the significant wave height in the presence of a strong background current.

ADCP Measurements

Pressure:

RDI ADCPs measure the pressure at the head of the instrument. The pressure is used primarily to measure the water depth above the instrument in order to ensure that subsurface measurements can be accurately transferred to the surface. It is also used as a redundant time series measurement for calculation of the non-directional wave parameters.

Range to Surface (Surface Tracking):

RDI ADCPs also measure the intensity of the sound that is reflected back to it (the Returned Signal Strength Intensity or RSSI) as a quality control parameter. The water surface will reflect sound much more strongly than the particles within the water, so analysis of the RSSI allows an accurate measurement of the range to the surface along each of the four beams. These four measurements of range are referred to as “Surface Tracking”.

Velocity Profiles:

ADCPs gather profiles of water velocity by measuring the Doppler shift of sound reflected from scatterers assumed to be passively following the flow. The measurements are range-gated into a series of bins along thee or more beams and then combined to infer the velocity profile encompassed by the beams. RDI ADCPs use a BroadBand measurement technique to transmit coded pulses which allows multiple measurements with each transmission. This technique achieves highly accurate measurements of the velocity profile over the entire water column at much higher data rates than can be achieved using other, simpler Doppler techniques.

Constructing the Array

The RDI ADCP Waves Array Technique involves creating an array of measurements at the surface by combining direct measurements of the surface with subsurface measurements that are transferred to the surface using the techniques described in equations (1)-(4). Fig. 13 is an illustration of the various measurements that are taken for the RDI ADCP Waves Array Technique:

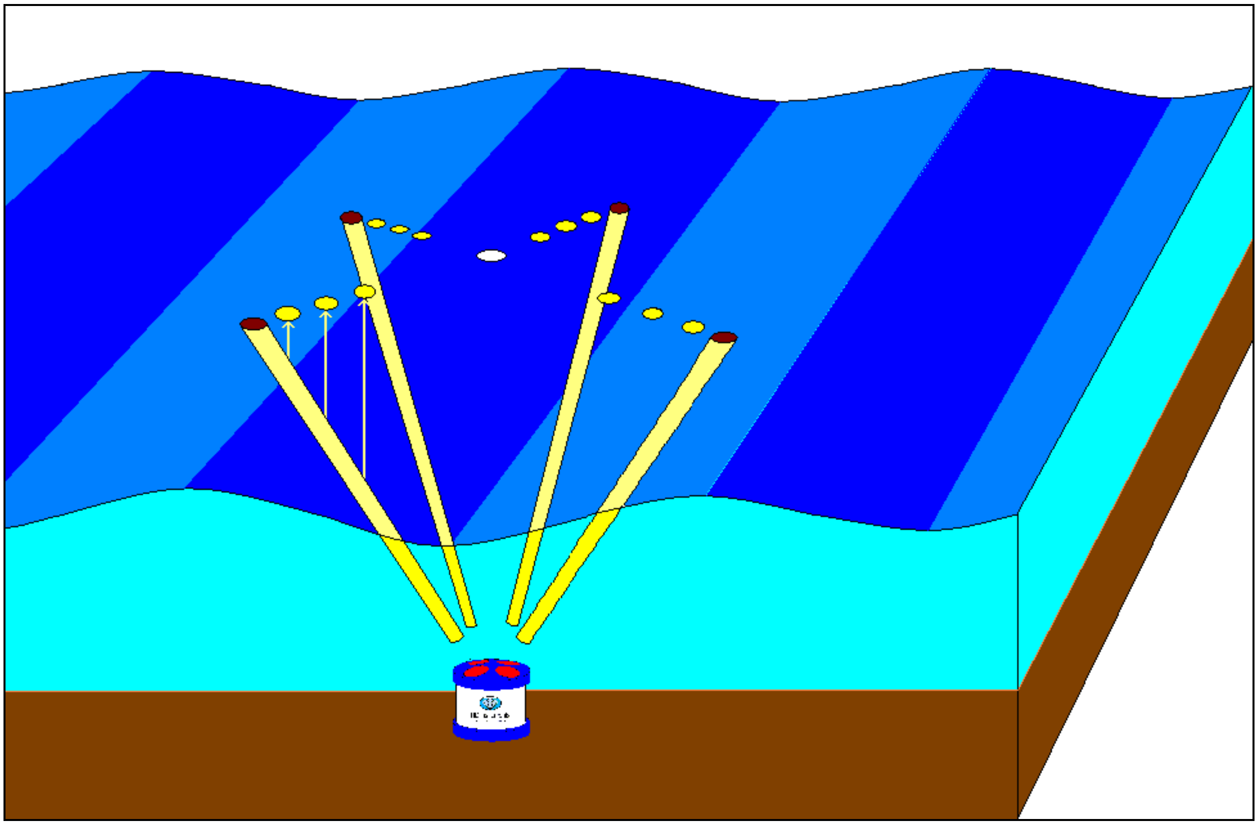

Fig: 13: Illustration of how the RDI Waves Array is constructed. Twelve measurements of velocity (three from each beam) are transferred to their equivalent surface velocity. The pressure measurement is transferred from the head of the instrument to the surface. There also four direct measurements of range to the surface.

Wave Parameter Measurements using the ADCP

Turning now to a discussion of how the wave parameters are determined using the RDI ADCP Waves Array Technique, it is again useful to consider the non-directional parameter measurements separately from the directional parameter measurement.

The Non-Directional Spectra

The bottom-mounted ADCP depicted in Fig. 13 enjoys the redundancy of three different methods to determine the non-directional spectra:

Pressure:

The ADCP is equipped with a pressure sensor whose primary purpose is to measure the water depth above the instrument, but it is also a time series of wave fluctuations. Calculating the non-directional spectrum

using the pressure sensor has the advantage of familiarity to most researchers. However, the pressure measurement is the deepest measurement made by the ADCP - which means that:
1) The measurement must be transferred the farthest distance to the surface, making it more susceptible to contamination by a background current (recall that deeper measurements in a background current result in a larger amplification during the transfer).
2) More importantly, the deeper the measurement, the lower the maximum frequency of resolvable waves (recall that the attenuation of wave energy with depth scales with the frequency)

The RDI Waves Array Technique reports the non-directional spectrum calculated from the pressure sensor measurement as a quality check on the other measurements. It is not the primary source for the non-directional spectra because it can not measure to as high a frequency as other techniques.

Surface Track:
The ADCP directly measures the range to the surface along each of its four beams, providing four more measurements of the non-directional spectra. This technique has the advantage of directly measuring the water level – no transfer function to the surface is required. This means that no decay with depth need be considered, and that the upper limit to the frequency of waves that can be resolved is solely due to the sampling interval. However, there are occasions, for example during glassy seas, where the sound glances off the surface without sending any signal back in the direction of the ADCP. Such cases are extremely rare, but they do happen. For this reason, the RDI Waves Array Technique treats the spectra derived from surface track measurements as a backup measurement to the primary technique.

Orbital Velocities:
The ADCP measures the along-beam velocities in a series of bins extending along each beam away from the head of the instrument. The BroadBand measurement technique allows accurate measurement of these velocities throughout the water column at very high data rates – typically 2 Hz. Because the velocities are so accurate, they can be transferred to their surface equivalent for calculation of the non-directional spectra. The RDI ADCP Waves Array Technique chooses three bins from each beam to transfer to the surface for a total of twelve independent measurements of the non-directional spectra. Choosing bins near the surface allows measurement of much higher frequency waves than can be measured at the depth of the pressure sensor. Also, unlike the surface track measurements, the velocity measurements remain robust under all sea states. For these reasons, the RDI Waves Array Technique chooses the orbital velocity measurements as the primary vehicle for calculation of the non-directional wave spectra. The non-directional spectra obtained from the other two measurements are reported as a quality check.

Though the ADCP is deployed on the bottom, it can measure higher frequency waves because it remotely measures the orbital velocities near the surface. Fig. 14 shows the approximate decay of the measurable wave energy that can be seen by an ADCP using near-surface orbital velocity measurements in comparison to the pressure sensor mounted to measure near the insturment. Note that a five second wave can be measured from an ADCP deployed as deeply as 90 m, compared to the 20 m maximum depth required for a pressure sensor.

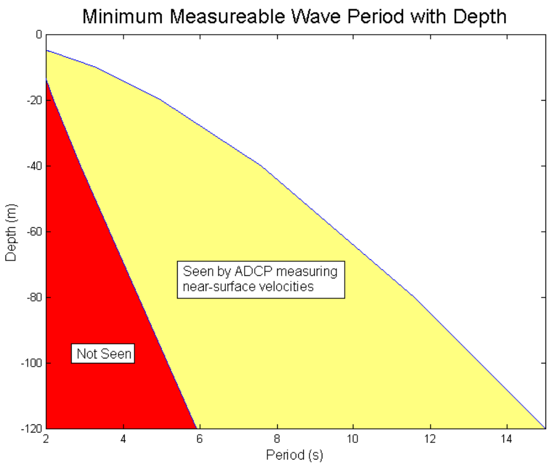

Fig. 14 The approximate decay of wave energy with depth that is resolvable by an ADCP using near-surface velocity measurements. The reported depth is the mounting depth of the ADCP, the yellow area is the limit of measurements for the pressure sensor which is located at the mounting depth (note: the upper bound of this area corresponds to Fig. 3). However, the ADCP measures velocity profiles to near the surface, which allows it to measure much higher frequency waves than can be seen at the mounting depth. Only the red area is excluded from the near-surface velocity measurements.

A standard non-directional spectra plots as output from the RDI ADCP Waves Array Technique software is shown below. Note that all three measurements agree very well, and that the high frequency cutoff decreases with the depth of the measurements, as expected.

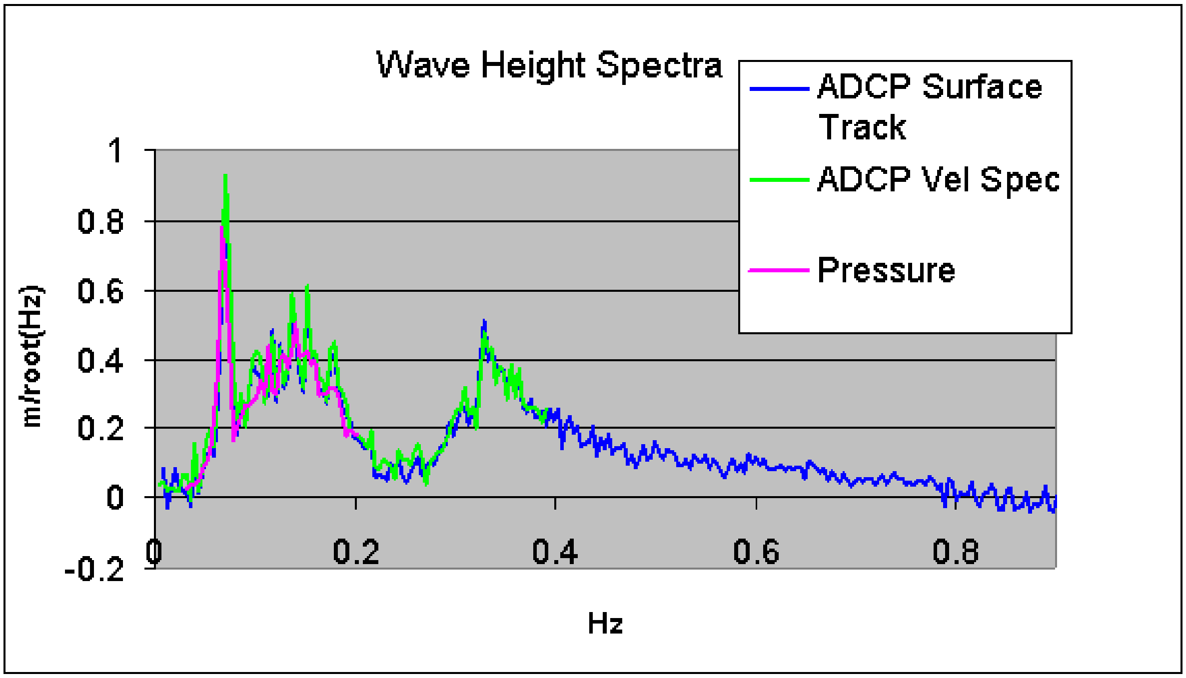

Fig. 15:A non-directional spectrum as output by the RDI ADCP Waves Array Technique. The spectra calculated by each of the three independent techniques agree very well. The high frequency limit decreases with the depth of the measurements, with the pressure sensor at the instrument head seeing the most significant limitations while the surface tracking measurements are limited only by the rate of data measurement.

Correction for Background Currents:

In the presence of strong background currents, the subsurface measurements will result in an overestimate of the wave height. This can be corrected using the profiled velocity data to create the weighted background velocity field to use in equation (4). Fig. 16 below shows non-directional spectra that were gathered in the presence of a large background current, and demonstrates the differing spectra that are obtained when that background current is neglected or included. Notice that the spectrum calculated from the pressure sensor is the most affected, which is due to the fact that the pressure sensor measures the deepest, and therefore has the most amplification applied to it.

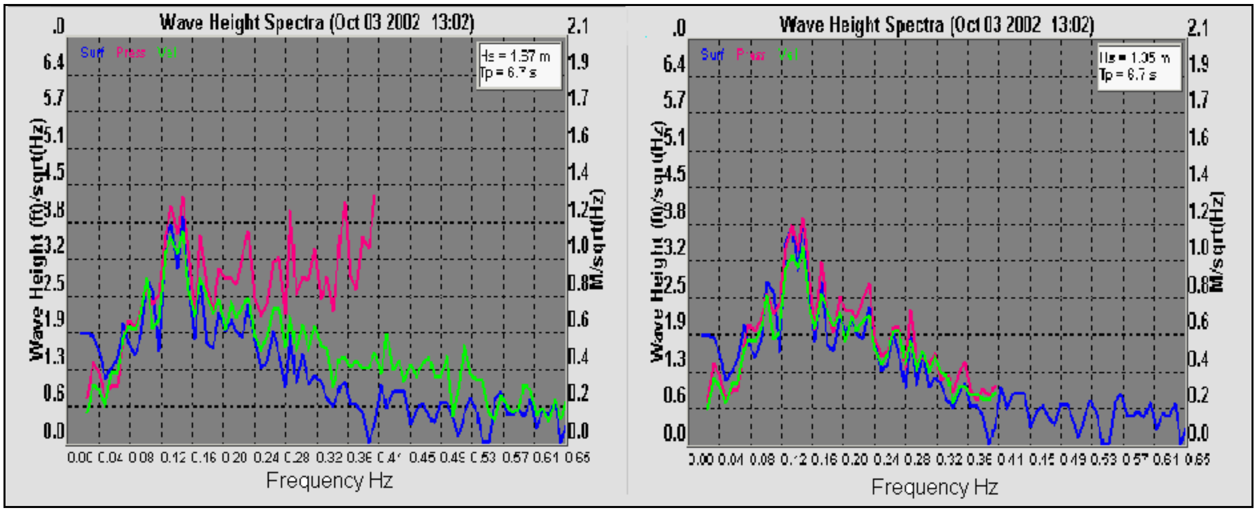

Fig. 16: Non-directional wave spectra gathered during a large background current. The panel on the left transferred the data to the surface values without correcting for the background currents, the panel on the right is the same data, but the transfer properly included the background currents.

The Directional Spectra

The twelve measured orbital velocities, after transfer to the surface, form an array which is then processed to determine the wave directions. The *H* function of equations (7) and (8) is calculated for the particular array geometry shown in Fig. 13. Equation (8) is then solved for each frequency and direction to determine the distribution of energy. Directional wave measurements are commonly shown as a polar plot as in Fig. 17. The direction is in degrees clockwise around the circle, and the frequency increases with range from the origin.

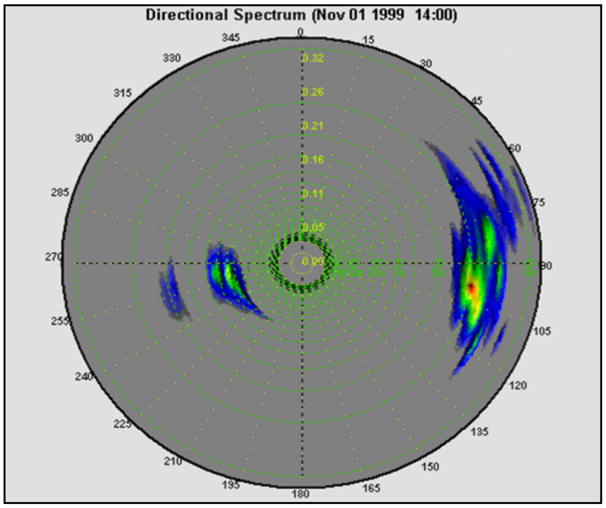

Fig. 17: Polar plot of directional wave energy. The frequency increases with distance from the origin.

Fig. 17 is showing two wave trains of different frequency arriving from different directions. This particular data was taken in area where there was a ten second swell incident from the West simultaneously with a four second wave (from a local storm) incident from the East.

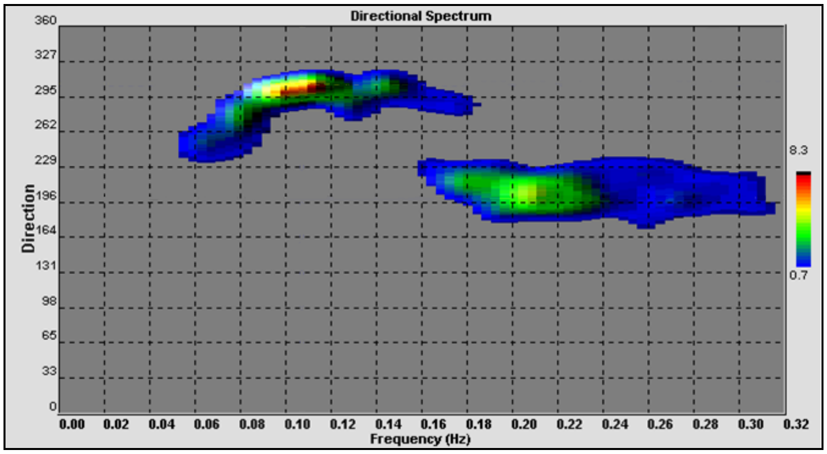

Fig. 18**: **Similar to Fig 17, except direction is plotted on the y-axis and frequency on the x-axis. Note the presence of two waves at 0.17 Hz propagating in different directions.

The real power of the RDI ADCP Waves Array Technique is that, as an array, it is capable of measuring waves of similar frequency propagating in different directions. Fig. 18 is similar to Fig. 17, except that is a Cartesian plot of the energy on direction vs. frequency axes. Note particularly in the center of the plot that the ADCP Waves Array Techniques is resolving two different wave trains with 0.17 Hz frequency (about six second period): one is incident from about 295 degrees and the other is incident from about 225 degrees. A PUV gauge deployed in the same seas reported a single wave direction of 260 degrees – where there was in fact no energy of any significance.

Summary

This primer was written with two goals in mind: the first was to simply describe the field of directional wave measurements. To recap: directional wave measurements are an attempt to statistically characterize wave energy in terms of its magnitude, frequency and direction. The magnitude and frequency are relatively easy to characterize from a single instrument, but care must be taken if subsurface measurements are used. Determining direction requires more than one measurement, and the greater the number of independent measurements the greater the resolution of the technique.

The second goal was to present the RDI ADCP Waves Array Technique. To summarize this technique it:

- Uses the beam geometry of a single, bottom-mounted ADCP to create a multi-element array of surface measurements.
- Uses three independent techniques to determine the non-directional wave parameters. The spectra obtained from the orbital velocities are considered to be the primary measurement. The spectra obtained from the pressure sensor and from the range to surface measurements are also provided, but for redundancy and quality assurance.
- Applies a Maximum Likelihood Method to a twelve element array of orbital velocities (after correcting to the surface) to determine wave direction. This allows the resolution of waves of similar frequency propagating in different directions, which ordinarily would require deploying several instruments.
- Uses the vertical profile of velocities to determine the properly weighted background current and exclude any such current from the transfer of the measured subsurface parameters to the surface.

The RDI Waves Array Technique is as easy to deploy as any single point instrument deployed to measure PUV. But through the use of BroadBand processing and the creation of an array, it is a far more capable instrument.

Further Reading

An excellent qualitative discussion of ocean waves, along with a number of entertaining sea stories, is provided by:

Bascom, Willard, 1980: *Waves and Beaches,* Anchor Books, Doubleday, New York, USA.

For the more mathematically inclined there are a number of excellent texts, including:

Kundu, Pijush K., 1990: *Fluid Mechanics*, Academic Press, San Diego, USA.
Lighthill, James, 1978: *Waves in Fluids*, Cambridge University Press, Cambridge, UK

For the mathematical details of the statistics presented:

Priestly, M.B., 1981: *Spectral Analysis and Time Series*, Academic Press Ltd., Great Yarmouth, UK

For more mathematical treatments of directional wave measurements there are number of resources including, but by no means limited to:

Dean, Robert G. and Robert A. Dalrymple, 1991: *Water Wave Mechanics for Engineers and Scientists*, World Scientific, Singapore

Longuet-Higgins, M.S., D.E. Cartwright and N.D. Smith, 1963: Observations of the Directional Spectrum of Sea Waves using the Motion of a Floating Buoy. *Ocean Wave Spectra*, Prentice-Hall, 111-136

Capon, J., R.J. Greenfield, R.J. Kolker, 1967: Multidimensional Maximum-Likelihood Processing of Large Aperture Seismic Arrays, *Proc. IEEE*, **55**, 192-211

Pawka, S.S., 1983: Island Shadows in Wave Directional Spectra. *J. Geophys. Res.,* **88**, 2579-2591

Oltman-Shay, J. and R. Guza, 1984: A Data Adaptive Ocean Wave Directional Spectrum Estimator for Pitch/Roll Type Measurements. *J. Phys. Oceanogr.*, **14**, 1800-1810

For more detailed exposition of the RDI Waves Array Technique:

Terray, EA, BH Brumley and B Strong, 1999: Measuring Waves and Currents with an Upward-Looking ADCP, *Proc. IEEE 6th Working Conference on Current Measurement,* IEEE, New York, 66-71

Strong, B, BH Brumley, EA Terray and GW Stone, 2000: Performance of ADCP-Derived Directional Wave Spectra and Comparison with Other Independent Measurements, *Proc. Oceans 00,* IEEE, New York.

Some comparative studies carried specifically on the RDI Waves Array Technique include:

Terray, EA, RL Gordon and BH Brumley, 1997: Measuring Wave Height and Direction Using Upward-Looking ADCPs, *Proc. Oceans 97,* IEEE, New York, 287-290.

Rørbæk, K and H Andersen, 2000: Evaluation of Wave Measurements with an Acoustic Doppler Current Profiler, *Proc. Oceans 00,* IEEE, New York

Shih, HH and B Strong, 2002: Laboratory Study of ADCP Wave Measurements, *Proc. OMAE 02,* ASME, New York