PARALLEL ALGORITHMS FOR MAXIMUM INDEPENDENT SET IN TRAPEZOID GRAPHS

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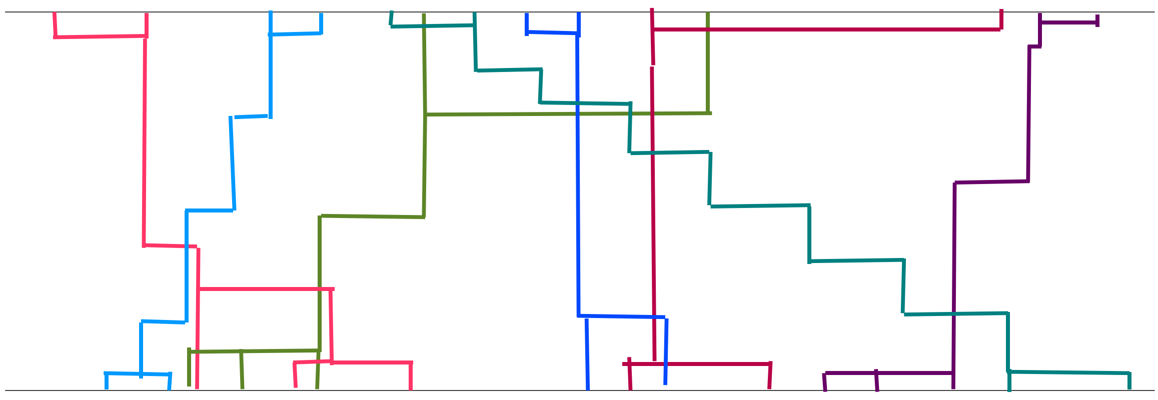

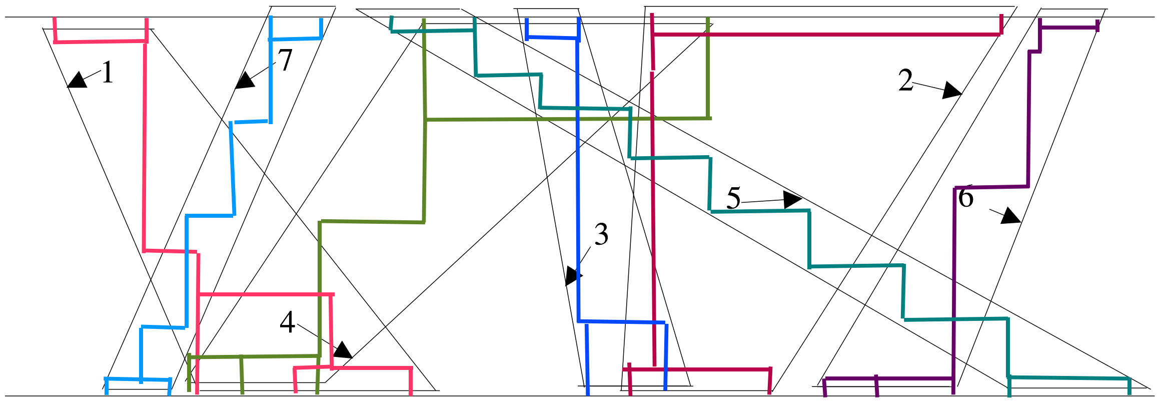
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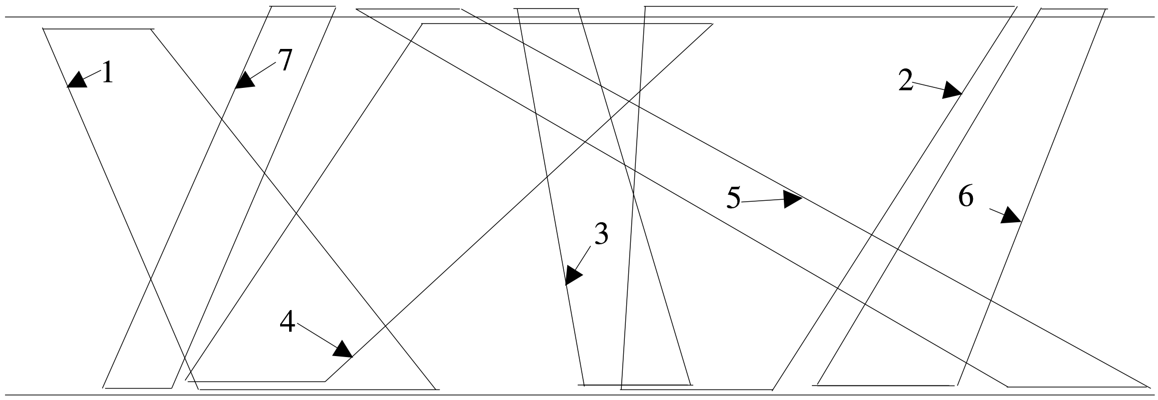
Definition: Trapezoid Representation

A double interval $I = (I_x, I_y)$ has **trapezoid** representation $T(I)$ defined as follows:

Given two parallel Real lines R_1 and R_2, if l_x and r_x denote the left and right end points of interval I_x on the real line R_1, and l_y and r_y denote the left and right end points of interval I_y on the real line R_2, then the trapezoid associated with double interval I is the convex hull of the four corner points l_x, and r_x on line R_1 and the points l_y, and r_y on line R_2.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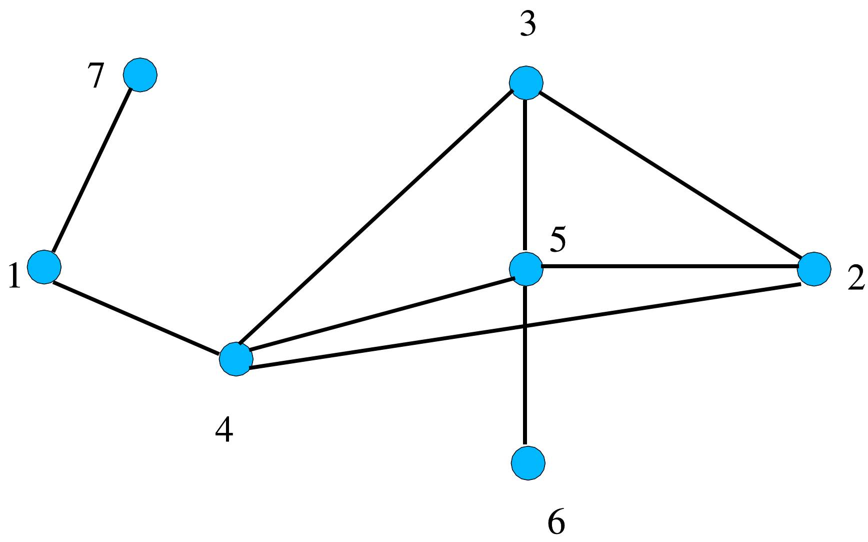
Definition: Trapezoid Graph

An undirected graph $G = (V, E)$ is a **trapezoid graph** if there is a mapping $f : V \to \mathcal{T}$ from the set of vertices V to a set $\mathcal{T}$ of trapezoids and that there is an edge $(x, y) \in E$ between vertex x and vertex y in the graph whenever the intersection between the trapezoid mapping x and the trapezoid mapping y is nonempty i.e., $f(x) \cap f(y) \neq \phi$.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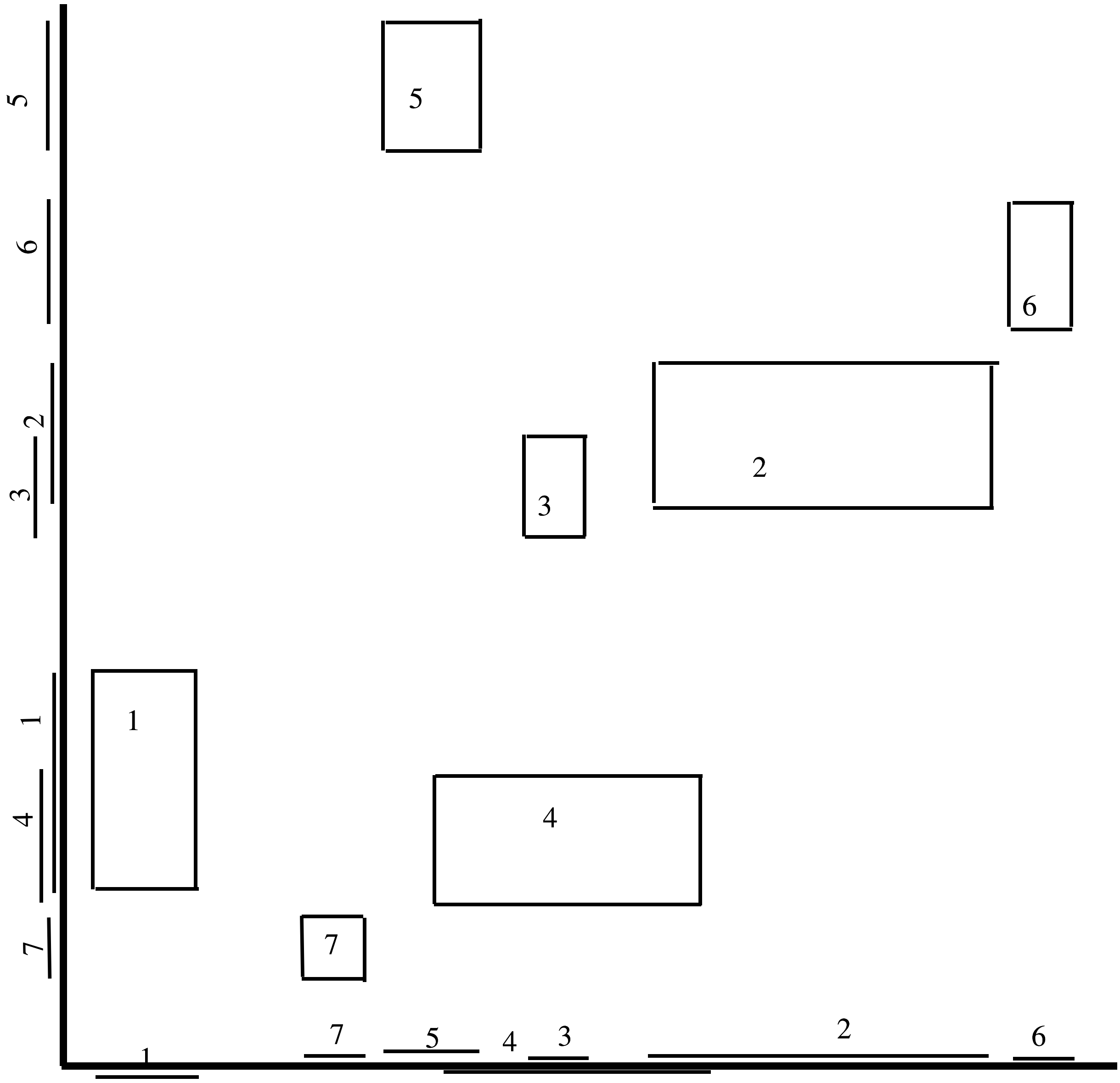
Definition: Box Representation

A double interval $I = (I_x, I_y)$ has **box representation** $B(I)$ in two dimensional rectilinear plane defined as follows:

If l_x and r_x denote the left and right end points of interval I_x on a real line R_1, and l_y and r_y denote the left and right end points of interval I_y on a real line R_2, then the lower left corner $ll(B(I))$ of the box $B(I)$ is at the coordinates (l_x, l_y) and the upper right corner $ur(B(I))$ of the box $B(I)$ is at coordinates (r_x, r_y).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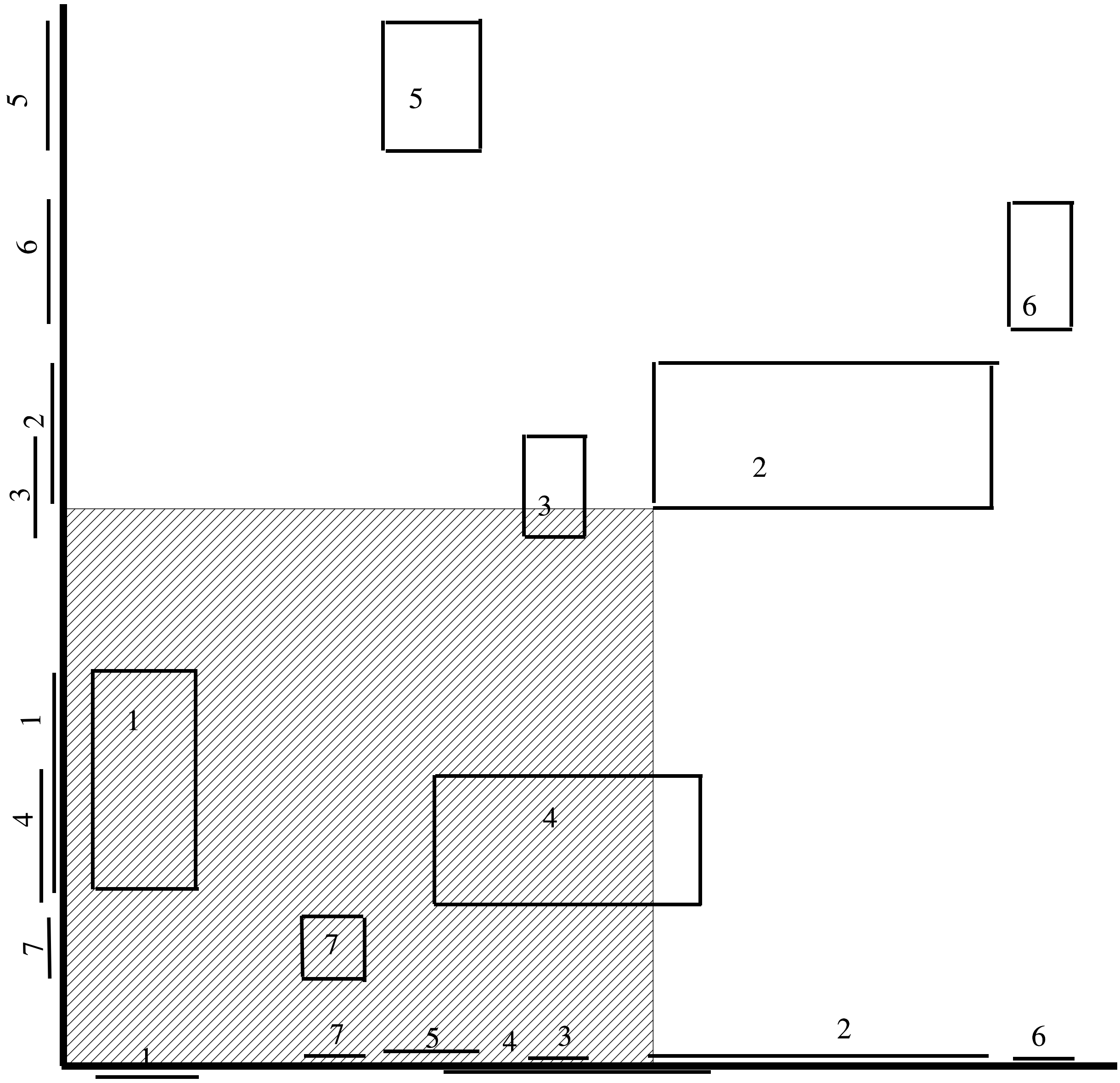
Definition: Box Domination

A box $B(y) = (ll(B(y)), ur(B(y)))$ **dominates** another box $B(x) = (ll(B(x)), ur(B(x)))$, denoted $B(x) < B(y)$, iff $ur(B(x)) < ll(B(y))$ (i.e., upper right corner $ur(B(x))$ of box $B(x)$ lies entirely to the left and below the lower left corner $ll(B(y))$ of the box $B(y)$).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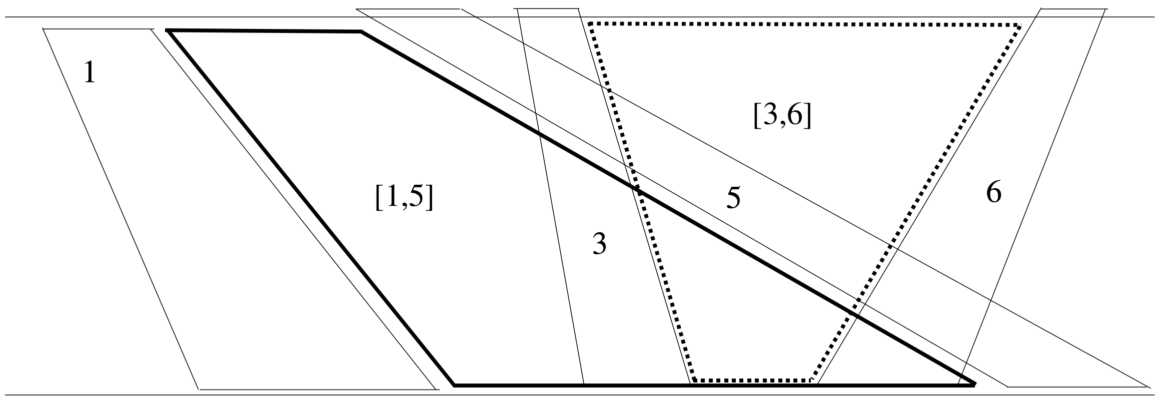
Definition: Box Direct Domination

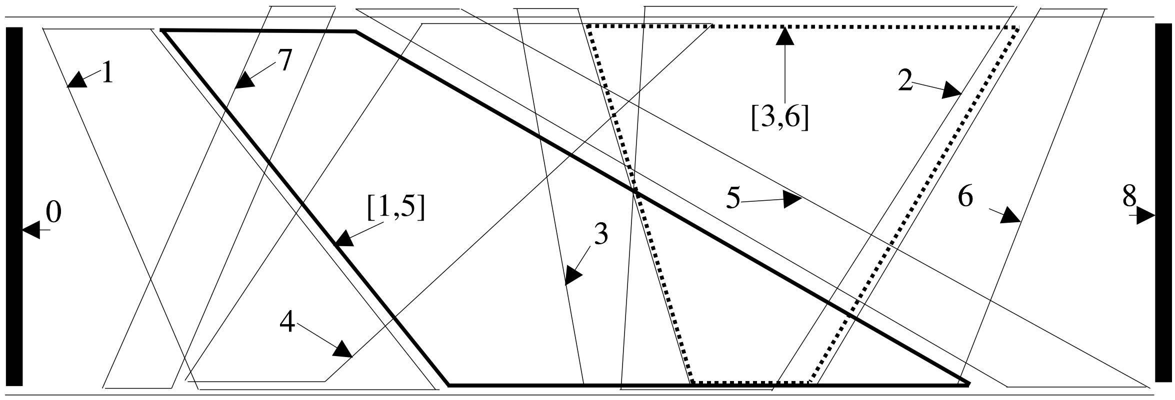
A box $B(y) = (ll(B(y)), ur(B(y)))$ **directly dominates** another box $B(x) = (ll(B(x)), ur(B(x)))$, denoted $B(x) << B(y)$, iff $B(x) < B(y)$ **and** there is no other box $B(k)$ such that $B(x) < B(k) < B(y)$.

Definition: In-Between Box, Trapezoid

An **in-between** box, denoted as $B_{[j,i]}$, is a box such that box B_i directly dominates box B_j (i.e., $B_j << B_i$) and the lower-left corner of in-between box $B_{[j,i]}$ is coincident with the upper-right corner of box B_j and the upper-right corner of in-between box $B_{[j,i]}$ is coincident with the lower-left corner of box B_i.

An **in-between** trapezoid, corresponds to an in-between box.


1
[1,5]
3
[3,6]
5
6


1
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[3,6]
0
[1,5]
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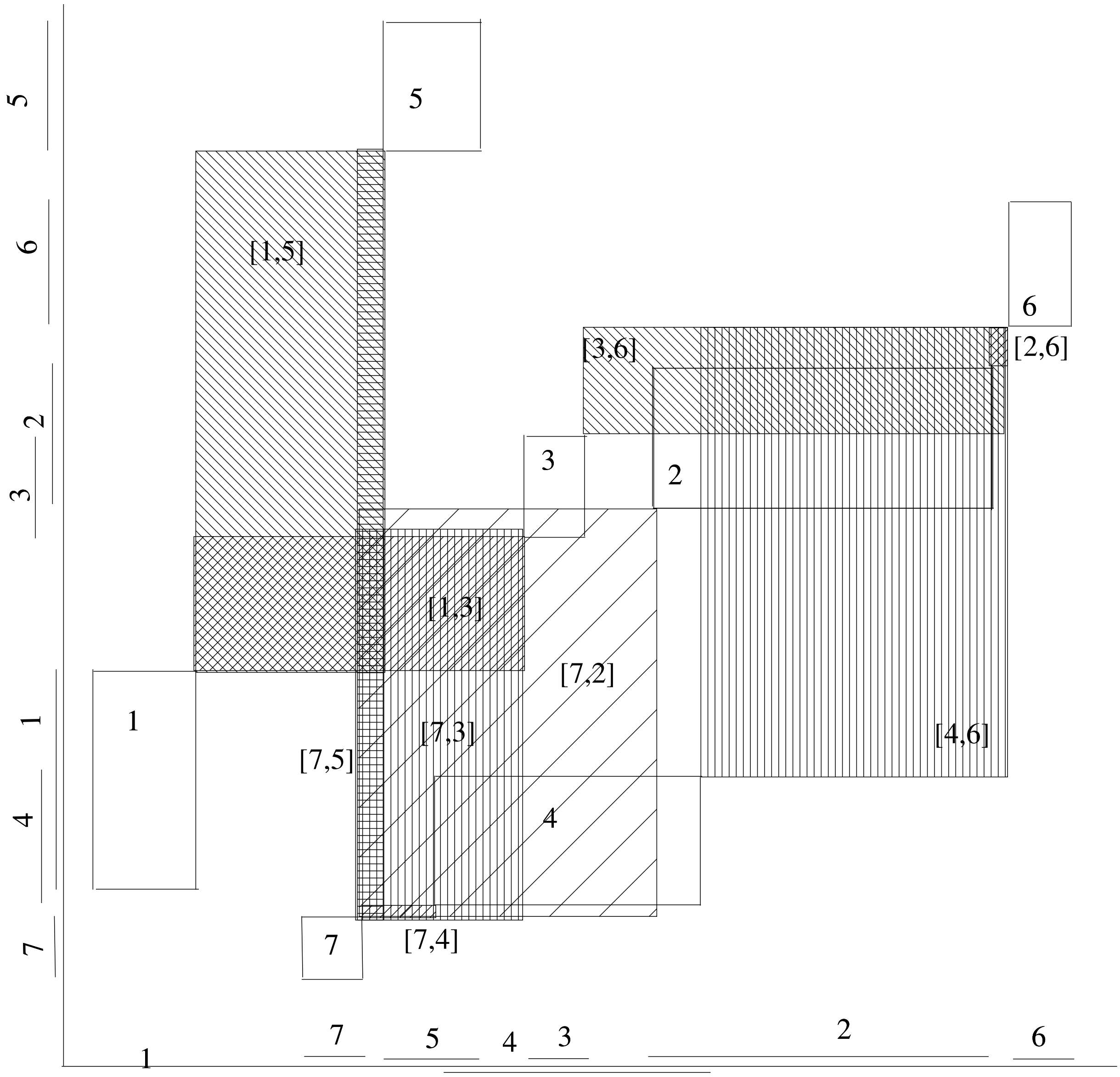

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[1,5]
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[3,6]
[2,6]
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2
[1,3]
[7,2]
1
[7,3]
[4,6]
[7,5]
4
7
[7,4]
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4
7
1
7
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Definition: Maximal Chain of Boxes

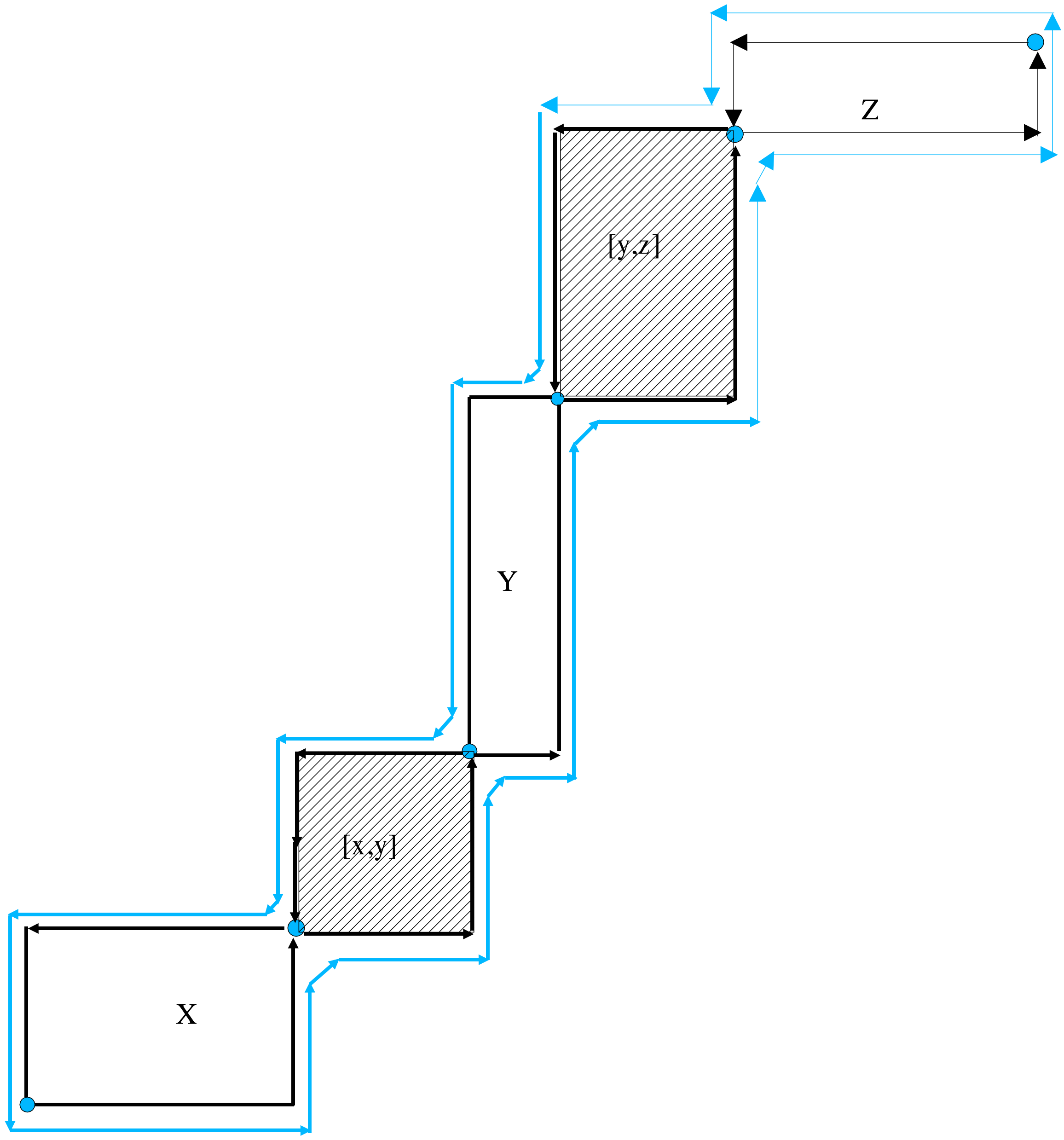
A **chain** of boxes is a sequence in which each box dominates all the preceding boxes in the sequence. The chain is **maximal chain** if no other box can be included in the sequence.

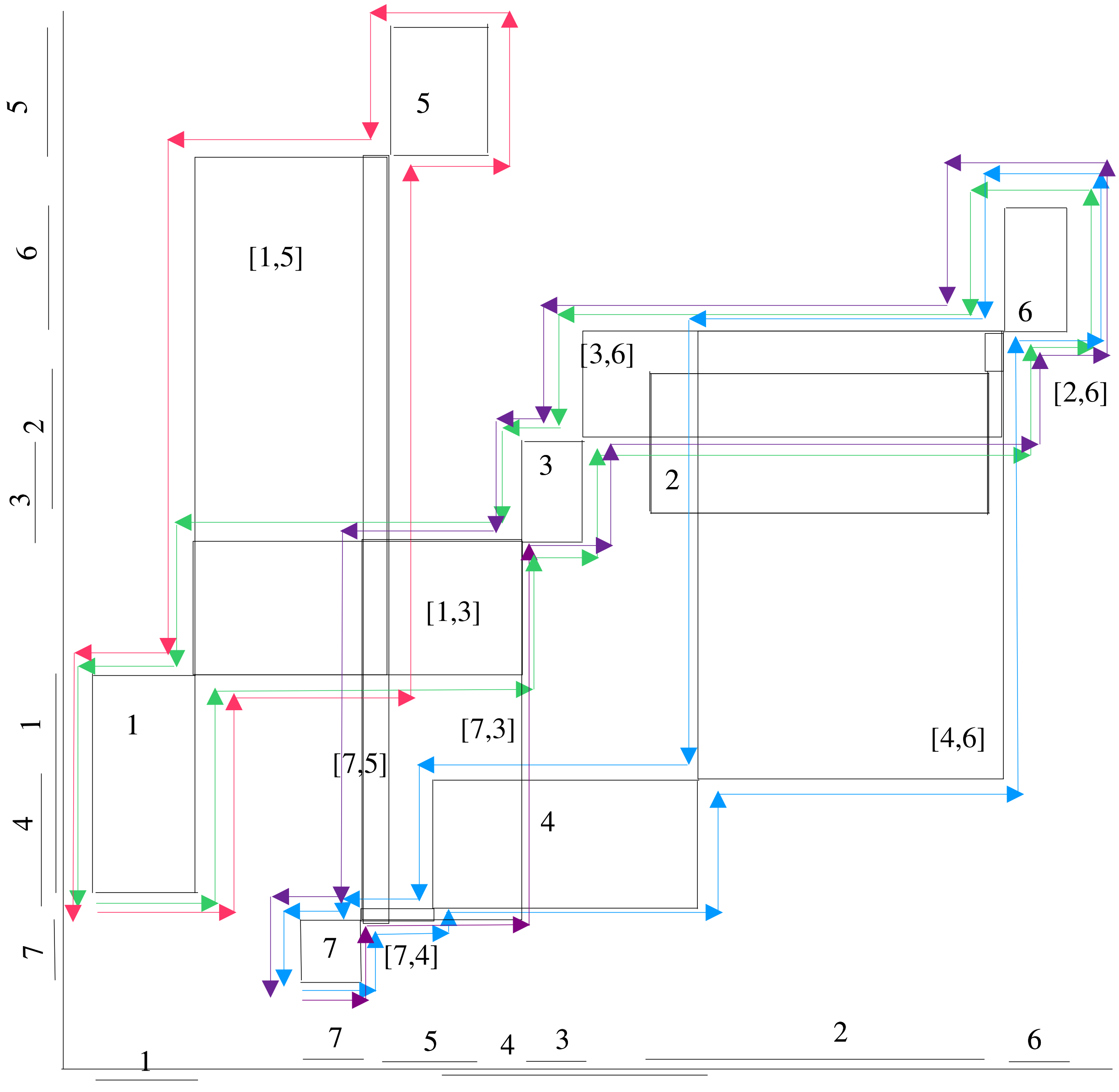
Lemma

A sequence of boxes $\sigma = (B_0, B_1, B_2, \ldots, B_k)$ is a maximal chain iff $B_j << B_{j+1}$ for all $j = 0 \ldots, k-1$ (every box in the sequence directly dominates its predecessor).

Lemma

Let $\mathcal{T}$ be the set of all trapezoids and $T_0, T_1, T_2, \ldots, T_k$ be any maximal chain with box representations $B_0, B_1, B_2, \ldots, B_k$ such that $B_0 << B_1 << B_2 << \ldots B_k$, then the alternate sequence of boxes and in-between boxes $B_0, [0,1], B_1, [1,2], B_2, [2,3], \ldots, [k-1,k], B_k$ corresponds to a trapezoid which spans the entire set $\mathcal{T}$


Z
[y,z]
Y
[x,y]
X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[1,5]
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[3,6]
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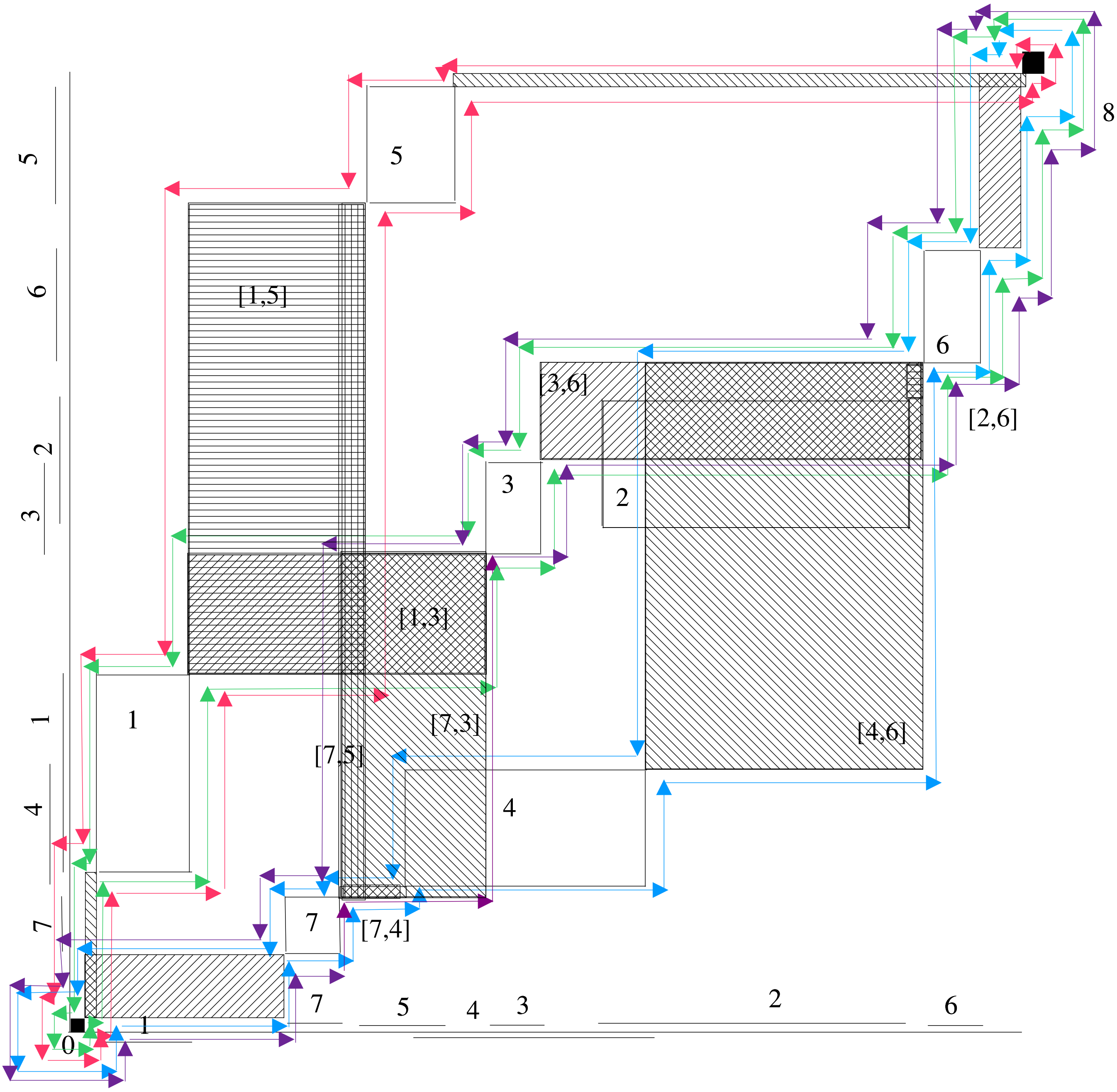

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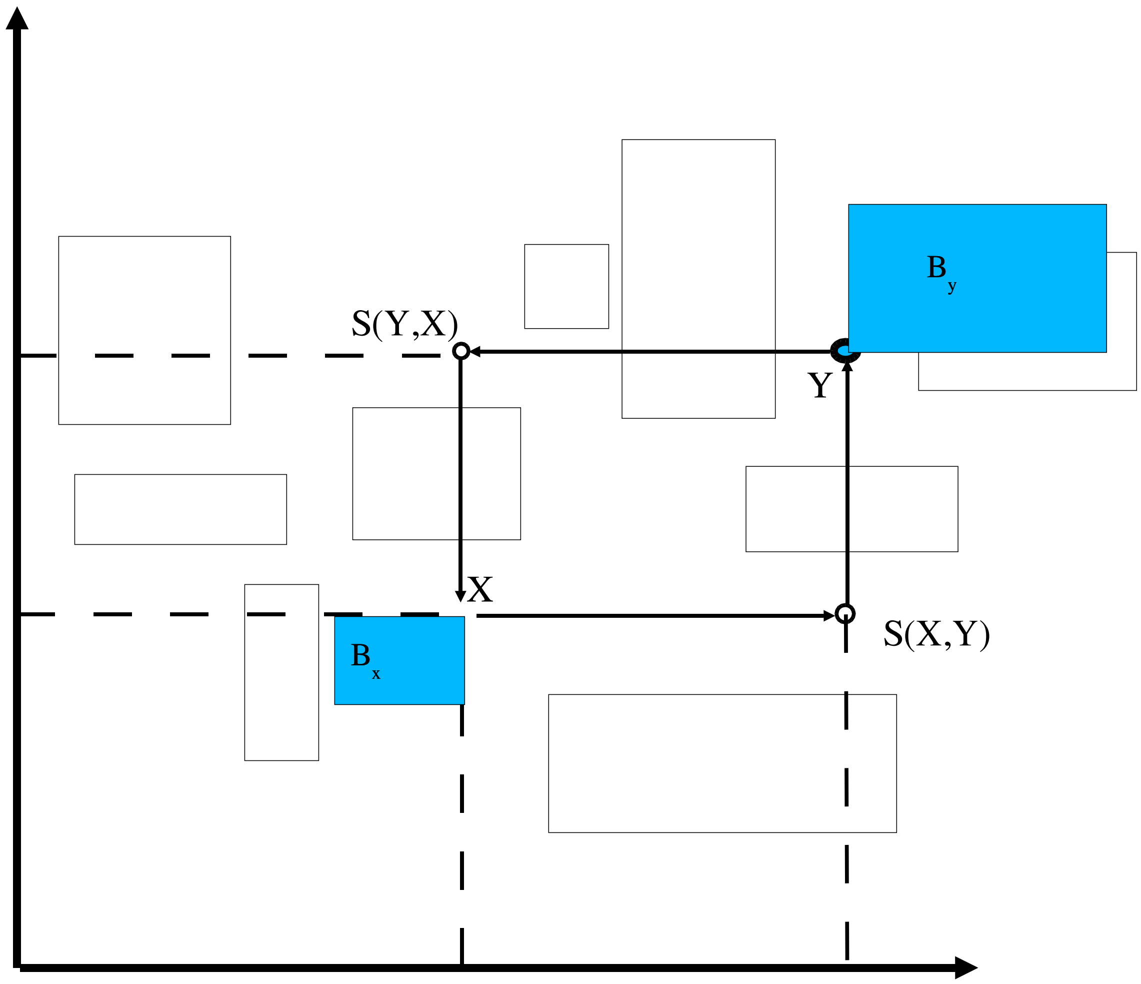
[1,3]
[7,3]
[7,5]
[4,6]
4
7
[7,4]
1
0
1
7
5
4
3
2
6

Algorithm Maximum Chain
(**input** $\mathcal{T}$:set_of_trapezoids;
output $\mathcal{I}$: Maximum Chain (Maximum Independent Set)
begin

- i **for all trapezoids** $T \in \mathcal{T}$ **in parallel** construct box representation B
- ii **for all lower left corner** points x in box representation **in parallel** call $CountOfLt(x)$
- iii **for all pairs** of corner points (x, y) **in parallel**
  mark the box pairs B_x, B_y
  if $Direct\_Domination(x, y)$ returns true
- iv **for pairs** of boxes (B_x, B_y) marked in step-(iii), define an in-between box $B_{[x,y]}$
- v **In parallel** Orient all boxes anti-clockwise
- vi **In parallel** Corner Stitch box B_x with in-between box $B_{[x,y]}$
- vii **In parallel** traverse chain of boxes from B_0 to B_k;
  count number of boxes in the chain from
  B_0 to B_k.
- viii **Output** the chain with maximum number of boxes as the Maximum Independent Set.

end.


B
y
S(Y,X)
Y
X
S(X,Y)
B
x

function Direct_Domination(x, y: corner points)
begin

if $(CountOfLt(y) - CountOfLt(S(x,y)) - CountOfLt(S(y,x)) + CountOfLt(x) == 0)$

/* the count of boxes inside the region defined by corners x and y is zero */

then

$B_x >> B_y$

/* Box with corner y directly dominates Box with corner x*/

end.

References

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Dagan, I., Golumbic, M.C., and Pinter, R.Y., Trapezoid Graphs and their Coloring, Discrete Applied Mathematics, 21 (1988), 35-46.

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